Endogenous Fertility and Social Security Reform in a Dynastic Life-Cycle Model

Radim Boháček and Volha Belush*
CERGE-EI, Prague, Czech Republic

June 2011

Abstract

This paper studies the effects of a fully funded social security reform on welfare, efficiency and inequality in a dynastic, life-cycle general equilibrium model with endogenous fertility. We compare the steady states of the pay-as-you-go and the fully funded systems when parents choose the number of children optimally. In the PAYG system, high-skill households save relatively more in assets than in children. As the high-skill households respond to the fully funded social security reform by having more children and investing less in assets, the average fertility increases and the aggregate capital stock falls. Our transition analysis shows that the privatization of the social security system is welfare improving and supported by the majority of the population.

JEL Keywords: E60 Macroeconomic Policy; H55 Social Security and Public Pensions; E62 Fiscal Policy, Public Expenditures, Investment, and Finance, Taxation; J13 Fertility

*We are grateful to the Czech Academy of Sciences (GAAV IAA700850702), the CERGE-EI Foundation and the GDN (RRC VII-55) for financial support. We received valuable suggestions from Albert Marcet, Evi Papa, Carlos Perez-Verdia, Hugo Rodriguez, Dimitrios Christelis, Tullio Jappelli, Marco Pagnozzi, Michal Myck, and seminar participants at Instituto de Análisis Económico (CSIC) at UAB, IHS in Vienna, Collegio Carlo Alberto in Torino, the CSEF in Naples, the University of Edinburgh, the DIW in Berlin, the SED Meetings, and the Spring Meetings of Young Economists in 2008. We are especially grateful to Luisa Fuster for providing us the parameters from her model. Serhiy Kozak provided excellent research assistance. This paper uses data from SHARELIFE release 1, as of November 24th 2010 or SHARE release 2.5.0, as of May 24th 2011. The SHARE data collection has been primarily funded by the European Commission through the 5th framework programme (project QLK6-CT-2001- 00360 in the thematic programme Quality of Life), through the 6th framework programme (projects SHARE-I3, RII-CT- 2006-062193, COMPARE, CIT5-CT-2005-028857, and SHARELIFE, CIT4-CT-2006-028812) and through the 7th framework programme (SHARE-PREP, 211909 and SHARE-LEAP, 227822). Additional funding from the U.S. National Institute on Aging (U01 AG09740-13S2, P01 AG005842, P01 AG08291, P30 AG12815, Y1-AG-4553-01 and OGHA 04-064, IAG BSR06-11, R21 AG025169) as well as from various national sources is gratefully acknowledged (see www.share-project.org for a full list of funding institutions). Address: Economics Institute, Academy of Sciences of the Czech Republic, Politickych veznu 7, 111 21 Prague 1, Czech Republic. Email: radim.bohacek@cerge-ei.cz.

1 Introduction

The pay-as-you-go social security system provides two important insurance roles. First, it partially substitutes for missing annuity markets during retirement. Second, it partially insures individuals against their labor productivity shocks. In this paper we analyze the social security reform in a detailed dynastic, general equilibrium model with endogenous fertility. We show that fertility choice together with intervivos transfers are able to replace these insurance mechanisms when the social security system is eliminated. The response of fertility and savings to social security leads to different aggregate allocations than those usually found in models where fertility is exogenous.

There is a large economic literature on the social security reform mostly in pure life-cycle models with heterogeneous agents and the assumption of exogenous fertility (Conesa and Krueger (1999), De Nardi et al. (1999), or Imrohoroglu et al. (1999)). These papers find that the elimination of social security brings large welfare, aggregate and distributional effects. The need for life-cycle savings leads to substantial increases of the capital stock (between 25-30%) and large general equilibrium effects. Agents are generally better off in the new steady state but the cost of a transition could be prohibitively high.

Our paper builds on the last important contribution to the social security literature, the dynastic models based on Fuster (1999), Fuster et al. (2003) and Fuster et al. (2007). These papers study the welfare effects of the social security reform in a general equilibrium model with overlapping generations of altruistic agents but with exogenous fertility. In the dynastic framework with a two-sided altruism, in addition to the life-cycle and insurance motives, individuals also save for bequest motives. Therefore, old agents do not necessarily have a lower marginal propensity to save than young agents. In this setting, Fuster et al. (2003) find that the current social security system with a 44% replacement rate crowds out only 6% of the capital stock. Thus the PAYG system has much lower effect on the aggregate saving rate in a framework with altruistic dynasties.

We examine these results in a detailed, dynastic life-cycle model where fertility is endogenous. In our model of two-sided altruism, the motivation for fertility comes from a combination of 'parental altruism' and 'old age security' approaches. In the first one, parents' utility depends on their own consumption, the number of children and the utility of each child (see the seminal paper by Barro and Becker (1989)). In the 'old age security' models (also 'children as investment' in Boldrin and Jones (2002) or Nishimura and Zhang (1992)), aging parents expect to be cared for by their children. These two approaches have very different qualitative and quantitative implications. In the Barro and Becker model, parents perceive their children's lives as a continuation of their own and the effect of government pensions on fertility is very small. The 'old age security' models are based on Caldwell (1982) theory of the fertility transition in which transfers from children to parents can generate high fertility choices. Institutions and government policies—such as the pay-as-you-go system—that eliminate the need for intervivos transfers within a family can reduce fertility. The social security tax can also reduce fertility among the borrowing constrained agents.

We find that the effects of endogenous fertility in social security models are large and important in their direction. In the PAYG system, low skill (low education) agents invest relatively more in terms of children while those with high skills (high education) invest relatively more in terms of assets. These savings-fertility differences lead to around 20% higher aggregate capital stock than in an otherwise identical PAYG steady state where fertility is exogenous. When the social security system is eliminated, parents can finance their retirement consumption either from savings or from the old-age support provided by their own children. Indeed, it is mainly the high skill individuals who shift from investment in capital to investment in the old-age support from children. Consequently, the fully funded reform (FF) increases fertility by 10.2% and decreases the capital stock by 8.3%.

In order to ascertain the welfare gains and political support for the reform, we simulate a transition path to the fully funded steady state. We find that the welfare gains from the reform are positive with more than 70% of the population supporting the reform. Finally, we analyze several PAYG and FF steady states with different assumptions on fertility and agents' heterogeneity to understand the important forces behind these outcomes. We show that it is the endogenous savings-fertility adjustment of individual households rather than various exogenous assumptions on fertility that is driving the main results of this paper.

These results indicate that increased fertility and intervivos transfers from children are able to partially replace the insurance mechanisms provided by the pay-as-you-go social security system. Therefore, models which impose exogenous fertility may undervalue the capital stock in the initial PAYG steady state by forcing the high skill agents to invest in children rather than savings. Consequently, these models may also predict the opposite direction of changes in the capital stock after the fully funded reform. This could lead to very different conclusions about the behavior of different groups of the population, aggregate outcomes, welfare gains, transition dynamics and political support for the social security reform.

The paper is organized as follows. The next Section describes the issues important for modeling endogenous fertility in a model with heterogeneous dynastic households. Section 3 develops the main model and equilibrium. Section 4 presents the calibration issues. Numerical results are shown in Section 5. Section 6 concludes. The Appendix contains the details on intervivos transfers.

2 Modeling Endogenous Fertility

This section develops a life-cycle model with endogenous fertility in the dynastic framework. For a simple exposition of modeling issues, we present a reduced one-period version of the dynastic model with individual households as in Fuster et al. (2003).

2.1 A Simple Dynastic Model with Exogenous Fertility

The decision unit is a household, composed of one father $f = 1$ and a fixed number of $\bar{s}$ sons.[1] In this model of two-sided altruism, there is no strategic behavior between the father and the children as they pool resources and maximize the same utility from a consumption per household member, $u(c/(f + \bar{s}))$. A dynasty is a sequence of households that belong to the same family line. We abstract here from life uncertainty.

The father is retired without any income. His sons of the next generation work for a wage w at a stochastic productivity shock z. Effectively, the sons provide a transfer to the father as old age security support. At the end of the period, the father dies and an exogenous number of children $\bar{n} = \bar{s}$ is born to each son. In the following period, the sons establish $\bar{s}$ new households in which each son becomes a single father with his own $\bar{n}$ sons. Savings a' is divided equally among the sons.

Households are heterogeneous regarding their asset holdings, a, and skills z. The Bellman equation for a household with a state $(a, f = 1, \bar{s}, z)$ is

$$v(a, \bar{s}, z) = \max_{c,a'} \left\{ u\left(\frac{c}{1+\bar{s}}\right) + \bar{s}^{\eta}\beta E[v(a', \bar{n}, z')|z] \right\},$$

subject to a budget constraint,

$$c + \gamma\bar{s}\bar{n} + \bar{s}a' \leq (1 + r)a + \bar{s}wz,$$

where a is assets, r is the interest rate, and γ is the cost of raising children.[2] Altruism in the sense of Barro and Becker (1989) is represented by a parameter η. The skill of sons in each household is partially correlated with the skill of their father through a Markov process.

The important point is that the discounted present value on the right hand side of the Bellman equation is, like in the Barro-Becker formulation, multiplied by $\bar{s}^{\eta}$. This multiplication incorporates the present discounted value of all the new $\bar{s}$ households into the dynastic value function. In this way the value function covers all households that have belonged, belong, and will belong to the dynasty. Note that while these new households have

[1] In this paper we abstract from two parent families. Jones et al. (2008) show that theories that explicitly distinguish between fathers and mothers are very similar to one-parent theories.

[2] In their model of exogenous fertility, Fuster, Imrohoroglu, and Imrohoroglu (2003) do not have the cost of children ($\gamma = 0$). Their altruism parameter $\eta = 1$.

the same amount of assets and composition, they might differ in their realized household idiosyncratic shock z'.

2.2 A Simple Dynastic Model with Endogenous Fertility

However, it is not possible to simply multiply the future value by s^η when fertility is a choice. The dynamic programming problem would not be well defined. Alvarez (1999) derives an equivalent formulation of the problem,

$$V(a,s,z) = \max_{c,a',n\geq 0} \left\{ u\left(\frac{c}{1+s}\right)(1+s)^\eta + \beta E[V(a',n,z')|z] \right\}, \tag{1}$$

subject to a budget constraint

$$c + \gamma sn + sa' \leq (1+r)a + swz. \tag{2}$$

This formulation does not work when the decision-making unit is an individual household rather than the economy-wide dynasty: As the future value is not multiplied by s^η, the dynasty does not take into account its division into the s new households. In other words, the discounted present value of utility of the dynasty only takes into account the utility of only one of the many households that follow in the dynasty line. All but one of the newly established households headed by former sons, who inherit the same amount of assets, are not valued.[3]

In order to study the behavior and allocations of individual heterogeneous households, we need to incorporate the splitting households back into the model up to a limit: If all these households were fully incorporated, the dimensionality of the state space would grow geometrically. The goal is to find an abstraction where 1) the budget constraint remains related to a single household, and at the same time 2) the *number* of relatives which a household considers in its decisions stays manageable and realistic. Put differently, in order to make utility increasing in number of children, we assume that households derive utility from the number of close relatives in other households but not from the utility of these members of the extended family.

For the latter condition we choose to only keep track of the newly established (i.e. separated) households and only for one generation when the direct relatives are alive. These households are headed by the new fathers who are brothers. Their number is b, equal to the number of sons s in the previous period. Being identical and coming from the

[3]For example, imagine two households: one with a single son and the other with five sons. Assume just here that they consume the same amount per household member, both have the same number of children per son, n, and the same savings *per son*, a'. Then the formulation in equation (1) would correctly value only the household with the single son. The other household with five sons would be substantially undervalued as only one out of the five sons would be be considered in the continuation of the dynastic functional equation.

same household, these brothers start their own families with the same bequest a' and the same number of own children (who might, however, draw different skills).

Thus we model a household whose head comes from a family that had b sons. Their number only multiplies the utility from consumption of the household members applying the same parameter of altruism η. Importantly, none of these separated households enters each other's period utility from consumption nor the budget constraint.

The state of an individual household is (a, b, s, z), where b is the number of sons in the previous period. The value function is

$$V(a, b, s, z) = \max_{c, a', n \geq 0} \left\{ u\left(\frac{c}{1+s}\right)(1+s)^{\eta} b^{\eta} + \beta E[V(a', s, n, z')|z] \right\},$$

subject to the above budget constraint in equation (2).

In other words, the economic unit is a single household. The existence of living, direct relatives has a positive externality. This externality is lost when the head of a household (father) dies. Everything else kept constant, large families have higher utility than smaller ones.

3 The Economy

This section describes the full overlapping generations model with endogenous fertility based on Fuster et al. (2003). The economy is populated by $2T$ overlapping generations. Each household consists of a father, $f \in F = \{0, 1\}$, sons $s \in S = \{0, 1, 2, 3, ...\}$, and the children each son decides to have, $n \in S$. For dynastic reasons discussed above, each household also values the fact that it comes from a family with $b \in S$ sons in the previous period. We will abbreviate the household composition as $h = (f, b, s, n)$. Because the full model has uncertain lifetimes, zeros indicate persons who are not alive. We assume that children die together with the sons and that when the father dies, the connection to his brothers is lost and the household ceases to value other households of the dynasty. All decisions are jointly taken by the $m = f + s$ adult members in a household.

The model period is 5 years. A household lasts $2T$ periods or until all its members have died. A timeline for a household is shown in Figure 1. The model age of the household is related to the age of sons. At $j = 1$ the sons are 20 when the father is 55 (model age $j = T + 1$). The father retires at real age 65 (model age j_R) and lives at most to age 90 which corresponds to model age $j = 2T$.

INSERT FIGURE 1 ABOUT HERE

In a life-cycle model with endogenous fertility we have to allow the sons to choose the number of children in the appropriate period of the life cycle. If the sons survive to model age $j_N - 1$ (real age 30), they choose the number of children born in the following period j_N. Children live in the same household in periods $j = j_N, \ldots, j_T$. After period T, the sons form s new households in which each of them becomes the single father in period $T + 1$. Each son takes his n children to his new household. Therefore, conditional on survival, during the first $j_N - 1$ periods an individual's life overlaps with the life of the parent and during the remaining $j = j_N, \ldots, 2T$ periods also with the lives of his own children. The fertility decisions of all households imply an endogenous population growth rate $\bar{n}$ that an individual household takes as given.

Households are heterogeneous regarding their asset holdings, age, composition and skills. The skill is revealed to each son in period $j = 1$ when he is aged 20 and enters the labor market. The skill is correlated with that of his father: it can be high or low, $z \in Z = \{H, L\}$, following a first-order Markov process

$$Q(z, z') = Prob(z' = j \mid z = i) \qquad i, j \in \{H, L\},$$

where z' and z are the labor abilities of the sons and their father, respectively. In other words, within a household, all offsprings have the same skill which might be different from that of the parent.

The skill is fixed for the whole life and determines an individual's age-efficiency profile, $\{\varepsilon_j(z)\}_{j=1}^{2T}$, as well as life expectancy through $\psi_j(z)$, the conditional probability of surviving to age $j + 1$ for an individual with an ability z who is alive in period $j = 1, \ldots, 2T$. We impose that at the terminal age $\psi_{2T}(z) = 0$. If all household members die, this branch of the dynasty disappears and their assets are distributed to all living adult persons in the economy by the government as lump sum accidental bequests.

3.1 Preferences

Individuals are altruistic, that is they care about their predecessors and descendants. As in Jones and Schoonbroodt (2007), adult household members in a dynasty care about their own consumption in the period, the number of children, and the utility of their children. In particular, 1) the utility of adult household members is increasing and concave in their own consumption; 2) parents are altruistic (holding fixed the number of children and increasing their future utility increases (strictly) the utility of the parent); 3) holding children's utility constant, increasing the number of children increases (strictly) the utility of the parent; and 4) the increase described in 3) is subject to diminishing returns. In addition, as discussed above, the household values the number of father's brothers in the separated households.

The household jointly maximizes utility from an equal amount of per-adult consumption,

$$U(c, h) = u\left(\frac{c}{f + s}\right)(f + s)^{\eta} b^{\eta},$$

where η is a parameter of altruism as in Barro and Becker (1989). The last term, b^η, represents the number of sons in the previous period.[4]

If the father is not alive, the link to other households in the dynasty is broken and the utility of the household with a state $h = (0, 0, s, n)$ is

$$U(c, h) = u\left(\frac{c}{s}\right) s^\eta.$$

The function u is a standard CES utility function, $u(\tilde{c}) = \frac{\tilde{c}^{1-\sigma}}{1-\sigma}$, for a per-adult consumption $\tilde{c}$. The preference parameters must satisfy the monotonicity and concavity requirements for optimization. We follow the standard assumption in the fertility literature where children and their utility are complements in the utility of parents (see also Lucas (2002)). Therefore, $u(c) \geq 0$ for all $c \geq 0$, u is strictly increasing and strictly concave and $0 < \eta < 1$. This implies $0 \leq \eta + \sigma - 1 < 1$ and $0 < 1 - \sigma < 1$. Jones and Schoonbroodt (2007) analyze these properties in detail.[5] In terms of Boldrin and Jones (2002), this model exhibits a cooperative care for parents by all siblings.

3.2 Production

We assume that the aggregate technology is represented by the standard Cobb-Douglas production function,

$$F(K_t, L_t) = K_t^\alpha (A_t L_t)^{1-\alpha},$$

where K_t and L_t represent aggregate capital stock and labor (in efficiency units) in period t. A_t is the technology parameter that grows at a constant exogenous rate $g > 0$. The capital stock depreciates at a rate $\delta \in (0, 1)$. Competitive firms maximize profits renting capital and hiring effective units of labor from households at competitive prices r_t and w_t, respectively.[6]

3.3 Government

The government in the economy finances its consumption G by levying taxes on labor income τ_l, capital income τ_k, and consumption τ_c. Social security benefits B are financed by a tax on labor income τ_{ss}. Finally, the government administers the redistribution of accidental bequests. All these activities are specified in detail below.

[4]For simplicity, we apply the same parameter of altruism. Observe that the brothers might not be actually alive because they draw their survival shocks independently.

[5]The other combination of parameters has different implications for the quantitative properties of the model. Children and their utility are substitutes if $u(c) \leq 0$ for all $c \geq 0$, u is strictly increasing and strictly concave and $\eta < 0$. A comparison of these two parameterizations will be the subject of our future work.

[6]In what follows, we drop the time subscripts.

3.4 Budget Constraint

Households are heterogeneous regarding their asset holdings, age, abilities, and composition. Denote (a, h, z, z') as the individual state of an age-j household, where a represents assets, $h = (f, b, s, n)$ is the household's composition, and (z, z') are the father's and sons' skills, respectively.

The budget constraint of a household with $m = f + s$ adult members is

$$(1 + \tau_c)(c + \gamma_j^g(n)e_j(h; z, z')) + (1 + g)a' = [1 + r(1 - \tau_k)]a + e_j(h; z, z') + m\xi, \qquad (3)$$

where c is the total household consumption, a' is savings of the whole household, ξ is the lump-sum transfer of accidental bequests, and τ_c and τ_k are the consumption and capital tax rates, respectively. In the calibration section we explain in detail the expenditures on children, $\gamma_j^g(n)$, a function of the number of children in period j.

The after tax earnings of the adult members is given by

$$e_j(h; z, z') = \begin{cases} fB_{j+T}(z) + s(1 - \gamma_j^w(n))\varepsilon_j(z')(1 - \tau_{ss} - \tau_l)w & \text{if } j \geq j_R - T, \\ [f\varepsilon_{j+T}(z) + s(1 - \gamma_j^w(n))\varepsilon_j(z')](1 - \tau_{ss} - \tau_l)w & \text{otherwise,} \end{cases} \qquad (4)$$

where $\gamma_j^w(n)$ represents a fraction of each son's working time devoted to n children in period j, τ_{ss} and τ_l are the social security and labor income tax rates, respectively. $B_{j+T}(z)$ are social security benefits, which depend on the father's average life-time earnings and wage in the retirement period.[7]

In all optimization problems below we impose a no-borrowing constraint, $a' \geq 0$.

3.5 Value Function at Age $j = 1, 2, ..., j_N - 2$

Let $V_j(a, h, z, z')$ be a value function of an age-j household with a assets, h members, and (z, z') skills. In periods $j = 1, 2, ..., j_N - 2$ there are no children yet so $h = (f, b, s, 0)$. The maximization problem is

$$V_j(a, h, z, z') = \max_{c, a'} \left\{ U(c, h) + \beta(1 + g)^{1-\sigma}\widetilde{V}_{j+1}(a', h', z, z') \right\},$$

subject to the budget constraint (3) and the after-tax earnings defined in (4).

The transition process for the value function is, due to life uncertainty,

$$\widetilde{V}_{j+1}(a', h', z, z') = \begin{cases} \psi_{j+T}(z)\psi_j(z')V_{j+1}(a', (1, b, s, n'), z, z') \\ \quad + \psi_{j+T}(z)(1 - \psi_j(z'))V_{j+1}(a', (1, b, 0, 0), z, z') \\ \quad\quad + (1 - \psi_{j+T}(z))\psi_j(z')V_{j+1}(a', (0, 0, s, n'), z, z') \\ & \text{if } f = 1, s > 0, \\ \psi_{j+T}(z)V_{j+1}(a', (1, b, 0, 0), z, z') & \text{if } f = 1, s = 0, \\ \psi_j(z')V_{j+1}(a', (0, 0, s, n'), z, z') & \text{if } f = 0, s > 0. \end{cases} \qquad (5)$$

[7] For a detailed definition see Fuster et al. (2003) and the calibration section. Variables are transformed to eliminate the effects of labor augmenting productivity growth.

While this transition is specified for all possible ages $j < T$, with no children in periods $j = 1, 2, ..., j_N - 2$, we impose $n' = n = 0$. Note again that sons share their survival uncertainty with their children and that the household stops remembering other relatives b when the father dies. Finally, if all members of the household die this branch of the dynasty disappears.

3.6 Value Function at Age $j_N - 1$

At the age $j_N - 1$, each son chooses the number of children that will be born in the following period. Being identical, all sons choose the same number of children, $n' \geq 0$,

$$V_{j_{N-1}}(a, h, z, z') = \max_{c,a',n' \geq 0} \left\{ U(c, h) + \beta(1+g)^{1-\sigma} \widetilde{V}_{j_N}(a', h', z, z') \right\}.$$

While the budget constraint (3) and after-tax earnings (4) are unchanged, the transition for the value function (5) now has $n' \geq 0$.

3.7 Value Function at Age $j = j_N, \ldots, T-1$

Children are born in period j_N and become a state variable in $h = (f, b, s, n)$ in periods $j_N, \ldots, T-1$. The value function is,

$$V_j(a, h, z, z') = \max_{c,a'} \left\{ U(c, h) + \beta(1+g)^{1-\sigma} \widetilde{V}_{j+1}(a', h', z, z') \right\},$$

subject to (3), (4), and (5). The number of children per son in the next period is $n' = n$ or $n' = 0$ if the sons die. Having children is costly in terms of goods, $\gamma_j^g(n)$, and working time, $\gamma_j^w(n)$.

3.8 Value Function at Age $j = T$

At the end of period T, a household transforms itself into s new households of the next dynastic generation in period $j = 1$. The father reaches the end of his life, each of the s sons becomes a single father and the children become n sons in each of the s newly established households (conditional on survival). Therefore, at $j = T$, the value function of a household $h = (f, b, s, n)$ with assets a and skills (z, z') is

$$V_T(a, h, z, z') = \max_{c,a'} \left\{ U(c, h) + \beta(1+g)^{1-\sigma} \sum_{z''} \psi_T(z') V_1(a', h', z', z'') Q(z', z'') \right\},$$

subject to a special period-T budget constraint,

$$(1+\tau_c)(c + \gamma_T^g(n) e_T(h; z, z')) + (1+g)sa' = [1 + r(1-\tau_k)]a + e_T(h; z, z') + m\xi,$$

the after-tax earnings (4), and a transformation to s new households with a composition

$$h' = (1, s, n, 0),$$

provided that the sons survive to form their own households. Otherwise, this branch of the dynasty dies off.

Note that the sons equally divide the household's assets. The skill of the sons in each of the new households is z'', correlated with the ability of the father, z'. Finally, there are no children in period $j = 1$ so $n' = 0$.

3.9 Stationary Recursive Competitive Equilibrium

Let $x = (a, f, b, s, n, z, z') \in X = (A \times F \times S \times S \times S \times Z \times Z)$ be an individual household's state. Denote $\{\lambda_j\}_{j=1}^T$ as age-dependent measures of households over x. Its law of motion for each $(a', f', b', s', n', z, z') \in X$ in periods $j = 1, ..., T-1$, is

$$\lambda_{j+1}(a', f', b', s', n', z, z') = \sum_{\{x:\, a'=a_j(x), n'=n_j(x)\}} \Psi_j(f', s'; f, s)\, \lambda_j(x),$$

where $\Psi_j(f', s'; f, s)$ is the probability that an age j household of type (f, s) becomes a type (f', s') in the next period. The number of sons from the last period is remembered $b' = b$ if the father survives and zero otherwise.[8] For a simpler exposition, $n_j(x) = 0$ for $j < j_N - 1$ and $n_j(x) = n$ for $j \geq j_N$.

The law of motion for the measure of age $j = 1$ households

$$\lambda_1(a', f', b', s', 0, z', z'') = \sum_{\{x: a'_j = a'_T(x)\}} s\, \Psi_T(f', s'; f, s)\, Q(z', z'')\, \lambda_T(x)$$

is now adjusted for the newly formed sons' households. Now $b' = s$ if the sons survive and zero otherwise.

Importantly, dynasties whose members die disappear from the economy. They are not artificially replaced by new households with zero assets and some arbitrary composition. In an equilibrium with endogenous fertility, new households established by the sons are so many that they not only replace the deceased dynasties but also deliver the desired population growth. Therefore, in the following definition there is no condition on new dynasties.

Definition 1 *Given fiscal policies $(G, B, \tau_l, \tau_k, \tau_c, \tau_{ss})$, a stationary recursive competitive equilibrium is a set of value functions $\{V_j(\cdot)\}_{j=1}^T$, policy functions $\{c_j(\cdot), a'_j(\cdot)\}_{j=1}^T$ and $n'_{j_N-1}(\cdot)$, factor prices (w, r), aggregate levels (K, L, C), lump-sum distribution of accidental bequests ξ, cost of children (γ_j^g, γ_j^w), measures $\{\lambda_j\}_{j=1}^T$, and a population growth rate $\overline{n}$, such that:*

[8] The transition probability matrices $\Psi_j(f', s'; f, s)$ for periods $j = 1, ..., T-1$ and $\Psi_T(f', s'; f, s)$ are straightforward and available upon request.

1. *given fiscal policies, prices and lump-sum transfers, the policy functions solve each household's optimization problem;*

2. *the prices (w, r) satisfy*

$$r = F_K(K, L) - \delta \text{ and } w = F_L(K, L);$$

3. *markets clear:*

$$\begin{aligned} K &= \sum_{j,x} a_j(x)\,\lambda_j(x)(1+\overline{n})^{1-j}, \\ L &= \sum_{j,x} [f\,\varepsilon_{j+T}(z) + s(1-\gamma_j^w(n))\,\varepsilon_j(z')]\,\lambda_j(x)(1+\overline{n})^{1-j}, \\ C &= \sum_{j,x} c_j(x)\,\lambda_j(x)(1+\overline{n})^{1-j}; \end{aligned}$$

4. *the measures $\{\lambda_j\}_{j=1}^T$ grow at the constant population growth rate, $\overline{n}$;*

5. *the lump-sum distribution of accidental bequests satisfies*

$$\xi = (1+r)\sum_{j,x} a_j'(x)\Psi_j(0,0;f,s)\,\lambda_j(x)(1+\overline{n})^{1-j};$$

6. *the government's budget is balanced*

$$G = \tau_k\, r\Big(K - \frac{\xi}{1+r}\Big) + \tau_l\, wL + \tau_c(C + C^g),$$

where C^g is the aggregate cost of children in terms of goods;

7. *the social security tax is such that the budget of the social security system is balanced*

$$\sum_{j=j_R}^{2T}\sum_{x} f\,B_j(z)\,\lambda_j(x)(1+\overline{n})^{1-j} = \tau_{ss} wL;$$

8. *and the aggregate feasibility constraint holds,*

$$C + C^g + (1+\overline{n})(1+g)K + G = F(K, L) + (1-\delta)K.$$

4 Calibration of the Benchmark Economy

The benchmark PAYG economy with endogenous fertility has the same replacement rate $\theta = 0.44$ as the U.S. economy. The capital share in the production function $\alpha = 0.34$ and the annual depreciation rate $\delta = 0.044$. We find that a parameter of altruism $\eta = 0.055$ and the annual discount factor $\beta = 0.988$ lead to the same steady state population growth rate as in the U.S. data ($\bar{n} = 0.012$) and a capital-output ratio 2.83. The coefficient of risk aversion $\sigma = 0.95$ satisfies the optimization restrictions (see Jones and Schoonbroodt (2007) for details). All parameters are presented in Table 1.

INSERT TABLE 1 ABOUT HERE

The conditional survival probabilities ψ are from Elo and Preston (1996). They imply that the life expectancy at real age 20 is 5 years longer for high skill individuals (college graduates) than that of low skill individuals.

4.1 Earnings, Social Security and Taxation

The efficiency profiles for low and high skills (see Figure 2) as well as their transition probabilities are calibrated to the profiles for college and con-college graduate males from data provided by the Bureau of Census (1991). As in Fuster et al. (2003), the targeted proportion of high-skill agents (college graduates) is 28% and the correlation between the father's and sons' wages to 0.4 as in Zimmerman (1992).

INSERT FIGURE 2 ABOUT HERE

Retirement benefits in the PAYG economy are related to an individual's lifetime earnings and are calculated according to the formula used by the Social Security Administration. The marginal replacement rate decreases with average lifetime earnings and is equal to 90% for earnings below 20% of the average earnings, to 33% for earnings above 20% and below 125% of the average earnings, to 15% for earnings above 125% and below 246% of the average earnings, and to zero for higher earnings. The social security tax $\tau_{ss} = 0.115\%$ clears the social security budget at 7.6% of GDP. In the steady states of the fully funded economy the replacement rate is set to zero.

The fiscal parameters are standard, taking values of 35% for the capital income tax and 5.5% for the consumption tax. Government consumption is set at 22.5% of the total output. We find that in the benchmark PAYG steady state with endogenous fertility, a labor income tax rate $\tau_l = 0.16$ satisfies the government budget constraint. All the reforms are computed to be neutral in government per-capita consumption.

4.2 Cost of Children

The *Report on the American Workforce* by the U.S. Department of Labor (1999) shows the average combined annual and weekly hours at work for married couples by presence and age of the youngest child. In 1997, the combined annual (weekly) hours were 3,686.6 (74.8) for couples with no children under age 18, 3,442.7 (70.4) with children aged 6 to 17, 3,545.0 (72.2) with children aged 3 to 5, and 3,316.5 (68.3) with children under 3 years. The labor force participation increases with time elapsed since the last birth, age of mother, education, and annual family income. It decreases only with the number of children. Table 2 presents these time costs, γ^w, adapted to this model's period structure as a fraction of a son's working time. We take the weekly measure as it is closer to the 3% of household time estimates in Boldrin, De Nardi, and Jones (2005).

INSERT TABLE 2 ABOUT HERE

Expenditures on children are taken from the 1990-92 Consumer Expenditure Survey of the Bureau of Labor Statistics. Estimates of the major budgetary components are for 12,850 husband-wife families with 1998 before-tax incomes between \$36,000 and \$60,600 (average \$47,900), controlling for income level, family size, and age of the younger child. Compared with expenditures on each child in a family with two children, households with one child spend on average 24 percent more on the single child, and those with three or more children spend on average 23 percent less on each child. Therefore, the USDA adjusts the expenditures by multiples of 1.24 and 0.77, respectively. Expenditures include housing and education. These numbers are comparable to the findings of Deaton and Muellbauer (1986) and Rothbarth (1943). The cost in the model γ^g is shown in the last column as a fraction of the average household before-tax income.

5 Results

The pay-as-you-go social security system provides two important insurance roles. First, it partially substitutes for missing annuity markets during retirement. Second, it partially insures individuals against their permanent labor productivity shock. On the other hand, the social security tax is distortive on the consumption-savings margin and is costly for the borrowing constrained agents. It also affects fertility decisions and influences the timing, direction, and amount of intergenerational transfers.

5.1 PAYG Steady States: Exogenous vs. Endogenous Fertility

In order to understand the effects of fertility choices on old-age insurance, we start our analysis by comparing two PAYG steady states with the same replacement rate $\theta = 0.44$ and the same U.S. population growth rate of 1.2%. In the first steady state the fertility decisions are exogenous and equal across different households; in the second, they are endogenous. All parameters are held constant across the two steady states.

In the exogenous fertility PAYG steady state we arrive at the desired population growth rate by imposing an equal fertility of 1.67 children born to each son in period j_N. A social security tax $\tau_{ss} = .115$ clears the government social security budget and $\tau_l = 0.168$ clears the government budget constraint at the the same level of government consumption per capita as in the benchmark PAYG steady state with endogenous fertility.[9]

[9]As these equilibrium labor tax rates are very similar in all steady states, they are not reported in the

INSERT TABLE 3 ABOUT HERE

In Table 3 we compare the aggregate allocations of these two steady states. It is apparent that differential endogenous fertility leads to very different allocations: the aggregate capital stock increases by 23.4%, output by 7.6%, consumption by 5.4%, and the capital-output ratio increases from 2.47 to 2.83. The equilibrium after-tax wage increases by 10.8%, the equilibrium interest rate falls by 0.9%.

Behind these different allocations are changes in households' fertility decisions. The bottom part of the table compares fertility and savings for different types of households based on their composition. The first four columns describe complete households subdivided into four categories according to the skills of the father and sons (for example, households where the father has high skill and sons low skills are denoted as HL). The next two columns show the results for households with only sons alive, of low (L) and high (H) skills. Complete households with both father and sons alive constitute around 77% of all households, those with only sons alive 21%, and those with only the father alive 2% (not shown in the tables). This implies that almost 63% of households are composed by only low skill members (46% of all households are LL), around 18% by only high skill members (HH households are 12% of the population), and 19% are either HL or LH type.

Among the complete households, the LL type increases fertility to 1.83 children per son while the fertility of the HH type falls by more than 30% to 1.11. When the sons live alone, the L types have on average the highest fertility, 1.85 children per son. All households with a high skill member reduce fertility and increase savings, while households of only low skill members increase both fertility and savings. In general, fertility decreases in skills (education) of the father and then of the sons.[10] The higher fertility in low skill households coincides with their relatively low time opportunity cost of having children, higher return on the PAYG benefits, and relaxed borrowing constraints from the higher after-tax wage (see more below). Although the fraction of high-skill individuals in the benchmark economy falls from 29 to 26 percent, the aggregate labor supply in efficiency units is almost the same due to a lower time cost of children borne by high-skill parents.

tables.

[10] In the 2004 Population Survey, low skill individuals have higher fertility than high skill individuals. In their survey of U.S. demographic history, Jones and Tertilt (2006) find that fertility is declining in education and income. The Children Ever Born measure (CEB, per woman per cohort aged 40-44 years) for women with a high school diploma is 1.943, and for women with a college degree 1.672. That is, high skill individuals have 13.9% lower CEB than those with low skill (when measured by the educational attainment of the husband, it is 12%). In our model, the difference for L and H lonely sons in the benchmark PAYG steady state is 13.0%. Jones and Tertilt (2006) also document a stable negative cross-sectional income elasticity of fertility of around -0.2 for U.S. cohorts born after World War I. In our benchmark PAYG steady state it is -0.32.

These results show that different households use different portfolios of old-age insurance tools. The assumption of equal exogenous fertility imposes a too high number of children on households with high skill members as well as underestimates the substitution effect between investment in assets and children. The outcome is that exogenous fertility leads to under-accumulation of capital. With endogenous fertility, the high skill households have fewer children, save more, do not dissave during retirement and are able to transfer their wealth into next generations (see more below and Figure 3). As children only partially inherit parents' skills, this wealth ends up over time in all household types of the dynasty and in the whole economy.

5.2 Fully Funded Reform with Exogenous Fertility

What does the assumption of imposing exogenous and equal fertility imply for the fully funded reform is shown in Table 4, where we show the aggregate allocations of the PAYG and FF steady states in which all sons have the same number of 1.67 children.

INSERT TABLE 4 ABOUT HERE

The FF reform eliminates the social security tax and benefits. Old age support from own children and savings are the only means for providing for consumption during retirement. Clearly, when the number of children is exogenous and equal across all household types, the only margin available for transferring resources into retirement is by increased savings. Thus in the FF steady state, the capital stock increases by 7%, the capital-output ratio increases from 2.47 to 2.59, together with output and consumption. The interest rate falls while the after-tax wage increases by 18.9%.

These changes are similar to other dynastic models with exogenous fertility.[11] The abolition of the social security tax and higher wages allow the low type agents to substantially increase savings (see LL and LH in Table 4). However, what seems to be more important is the effect on the accumulation of capital under exogenous and endogenous fertility. Comparing the change in the aggregate capital level due to the privatization of social security (7%) to that in Table 3 (more than 23%) suggests that in the dynastic models, the fully funded reform has smaller implications for accumulation of capital than just allowing for fertility choice in the PAYG system itself.

[11] In Fuster et al. (2003) the FF reform increases capital stock by 6.1%, output by 1.8%, consumption by 1.1%, and the capital-output ratio increases from 2.48 to 2.59. This increase in the capital stock is much lower than in the pure life-cycle models, with proportionately smaller general equilibrium effects (see discussion below).

5.2.1 Transition Analysis: Welfare Gains and Political Support

A proper analysis of welfare gains from privatizing social security must include the transition path to the new steady state. We compute the compensating variation in consumption to each household that equates the economy-wide average expected discounted utility in the PAYG steady state to that on the transition to the FF steady state.

The middle part of Table 4 shows that the average welfare gain from transition is +2.1%. In general, households with a low skill father benefit more than the other households. The LH households are much better off as the forgone social security benefits were small and the sons paid higher social security taxes. On the other hand, the HL household had the largest return on social security taxes as the father received larger benefits and sons paid lower taxes. Obviously, households where only sons are alive are better off while those with only father alive are worse off.[12]

At the bottom of the table we show the welfare gains for complete households (with both father and sons alive) of different age. The gains are all positive but not monotone, decreasing from age 20 to age 30 and then increasing until retirement. Complete households that gain the most are the age-20 households whose contribution to the system are smaller and potential gains large. The age-30 household suffer the most because these are the households in which the father has just retired and lost all benefits to which he had contributed. Later the gains increase as the father ages (probability of his survival falls) and sons' incomes increase.

Finally, Table 4 displays almost a full support for the reform except for the households where the sons have died and a few HL type households. The majority support for the reform is also found in Fuster et al. (2007). On the other hand, Conesa and Krueger (1999) report only 21% to 40% support for their uncompensated reform in a pure life-cycle model.[13] It is apparent that the dynastic framework with two-sided altruism allows for support and compensation of the transition costs within families and across generations. At the same time, the FF reform is not Pareto efficient nor it satisfies the concepts of $\mathcal{P}-$ and $\mathcal{A}-$efficiency in Golosov et al. (2007).

5.3 Fully Funded Reform with Endogenous Fertility

How much of a difference does endogenous fertility make for the fully funded reform can be seen from the main model of this paper presented in Table 5.

[12] We also compare welfare of sons' newly established households in both steady states: the gains are positive for all household types except the lonely fathers. Compared to the transition gains, the steady state gains are typically 1.1-1.2% larger.

[13] In a related paper Conesa and Garriga (2008) propose to eliminate these transition costs by issuing bonds that will be repaid from the future efficiency gains.

INSERT TABLE 5 ABOUT HERE

The FF reform internalizes the fertility decision with respect to the old-age support. Consequently, the overall fertility increases by 10.2 percent, resulting in a 1.5% population growth rate.[14] All households increase fertility except for the L sons living alone. The reform substantially increases the fertility of households with high skill fathers (21.2% for HL and 57.3% for HH households, respectively), with the HL households becoming the most fertile at 1.89 children per son. Fertility differences by skill decline to 2.3% (H vs. L), 5.3% (LH vs. LL), and 8.5% (HH vs. HL).[15]

As the high skill households shift from saving in assets to saving in children, in the FF steady state the capital stock *falls* by 8.3% and the capital-output ratio falls from 2.84 to 2.58. Households with a high skill father decrease their asset holdings by more than 20 percent. These are the households whose fertility increases the most. On the other hand, households with a low skill father slightly increase their savings. The only exception is again the lonely sons who lower their fertility as well as assets. Overall, the FF reform reduces both fertility and wealth differences across different household types.

Other models with endogenous fertility (Boldrin and Jones (2002) and Boldrin et al. (2005)) show a similar fall in the capital-output ratio when social security is eliminated.[16] Importantly, the changes in the capital stock are opposite to models where fertility is exogenous. Recall that dynastic models by Fuster et al. (2003) and Fuster et al. (2007) report an increase of the capital stock by 6.1% and 12.1%, respectively. Pure life-cycle models of Conesa and Krueger (1999), De Nardi et al. (1999), or Auerbach and Kotlikoff (1987), all suggest an increase of around 30%. Imrohoroglu et al. (1999) report that capital stock increases by 26% and Storesletten et al. (1999) by 10% to 25%. These large changes in the capital stock are driven by the forced savings imposed on high skill households in assets rather than children as well as by the underestimated capital stock in the original PAYG steady state with exogenous fertility (see Table 3).

[14] This is consistent with empirical evidence surveyed by Boldrin et al. (2005) on a cross-section data from 104 countries, who show a strong negative relationship between the total fertility rate and the size of a country's social security and pension system. Boldrin and Jones (2002) find that the Caldwell-type models based on the old age security motive account for between 40% to 60% of the observed fertility differences between the United States and other developed countries or in the United States over time. In their model, the increase in fertility due to the fully funded reform is 20-25%.

[15] Jones and Tertilt (2006) document that over the last 150 years of the U.S. demographic history, there has been a decrease in fertility inequality, especially with respect to income. The difference from the top to the bottom of the income distribution of fertility has been falling, from around 1.6 CEB for the 1863 cohort to a quarter of a child by the 1923 birth cohort. Since then it has stabilized.

[16] In Boldrin and Jones (2002), an increase in the social security from 0 to 10% of GDP leads to an increase in the capital-output ratio from 2.2 to 2.4, and to a reduction of TFR from 1.15 to 0.9. The Barro and Becker model predicts a decrease in the capital-output ratio.

The higher fertility of high-skill households leads to an increase in the efficiency labor supply by 5.8%. These changes in labor and capital inputs offset each other for only a small 0.8% increase in output. Thus the general equilibrium effects seem to be smaller than those found in related models. The average consumption decreases as higher fertility implies higher cost of children. Finally, the FF reform also improves the dependency ratio of the retired to working population from 18.7% to 17.5% (despite the increased longevity in the population).

5.3.1 Life-Cycle Savings

In order to document forces that are important for these savings-fertility decisions, Figure 3 shows the life-cycle accumulation of assets by complete households for the PAYG and FF steady states. The accumulation of wealth culminates in period $j_N = 4$ when children are born. In the consequent periods, the cost of children and father's retirement drive down the average wealth for all household types.

INSERT FIGURE 3 ABOUT HERE

Notice that households with a low skill father save less and leave lower bequests than those with a high skill father. In the PAYG steady state, assets of high skill fathers are mostly transferred to sons' new households. Especially in the HH households assets are almost fully bequest: social security benefits allow these households not to dissave at the end of the life cycle. On the contrary in the FF steady state, the HH and HL households use their assets for consumption in the retirement periods. As these types also substantially increase fertility, their bequest per son is even lower. This means that the lower capital stock in the FF steady state comes not so much from the ability to accumulate capital during the pre-retirement periods but rather from a different usage of the capital stock during retirement. The PAYG benefits allow households with a high skill father to transfer wealth across generations: If a household's budget constraint permits, assets are used as a partial insurance against a low realization of skill in future generations.

Consequently in the FF steady state, assets are not so persistently accumulated across generations and wealth inequality decreases. The Gini coefficient of wealth inequality is 0.48 in the PAYG steady state while in the FF steady state it is 0.45. In earlier papers, De Nardi et al. (1999) and Fuster (1999) find similar changes.

Overall, life cycle savings dominates in the PAYG system in the poor households while savings for bequest dominates in the wealthy households. Privatization of the social security system tends to eliminate the bequest-motivated savings as wealthy households invest in children for old-age support and insurance. In other words, resources that were used for bequest motives in the PAYG system are now used for raising children and consumption during retirement.

5.3.2 Intervivos Transfers

The shift towards saving in children should also be reflected in intervivos transfers that allow for consumption smoothing within households.[17] The definition of how these transfers are computed follows Fuster et al. (2003) and is described in the Appendix. A person in a household needs a transfer from other members of the household if he cannot cover his share of consumption expenditures from his own income and savings. It is assumed that the father holds all the assets of a household in period $j = 1$. In Figure 4, a positive number is the net transfer from the father to one son while a negative number is the net transfer from one son to the father, both as a fraction of the average income per household member. The full line represents transfers in the PAYG social security system and the dotted line in the FF system.

INSERT FIGURE 4 ABOUT HERE

Four main results stand out. First, in the FF system the sons always support their father during his retirement. Second, in the PAYG system the high skill fathers support their sons while the opposite is true for the low skill fathers. Third, the transfers are small in the first three or four periods: both fathers and sons are almost self-sufficient in their total incomes. The father supports his sons at least in the initial periods because he holds all assets in period $j = 1$ and is, at the same time, at the peak of his life-cycle earnings. The PAYG system contributes to these positive transfers as the father compensates the sons for social security taxes the latter pay. Fourth, the public transfer system of the PAYG system is replaced by the FF private transfers: the amount of intervivos transfers more than doubles.

Altogether, these changes suggest that increased fertility in the FF steady state is able to (partially) replace the old-age insurance provided by the PAYG benefits. For all complete household types the intervivos transfers represent a significant fraction of a household's income and increase over the retirement period. In poor households, they start immediately with retirement while in wealthy households, they start later but are never positive like in the PAYG system.[18] Given the large amount of these transfers, it

[17]For an interesting survey of intergenerational transfers in Europe, see SHARE (2005).

[18]In Fuster et al. (2003), transfers in the FF system go in the same direction. In their PAYG system, fathers support their sons in the first three periods while sons support the fathers in the last four periods (only in the HL case the transfers are always positive). This is likely because the sons have higher incomes as children are cost free and all household types have the same number of sons.

is plausible that poor households would like to increase fertility more if they were not borrowing constrained.

Data from the Survey of Consumer Finances show that about 75% of transfers go from parents to children (see Gale and Scholz (1994)). In our PAYG steady state, only 56% of intervivos transfers is from fathers to sons (if we add bequests, the amount of transfers increases to 74%). However, the FF reform reverses the intervivos transfers within the family: on average, more than 90% of transfers go from children to parents.

5.3.3 Transition Analysis: Welfare Gains and Political Support

Overall, the average welfare gain from transition is +0.5%. For reasons discussed above in the case of exogenous fertility, the households with a low skill father benefit more than the other households. These fathers now receive large transfers from their sons who receive larger after-tax incomes after the reform.

The LL and LH households experience a welfare gain while the HL and HH a welfare loss. The wealthier households tend to lose because they substantially increase their number of children during the initial periods. These associated costs are large before the household size stabilizes at new levels in the next generations. Only the the LH types are better off regardless of their age.[19]

Table 5 shows that 73% of the population supports the reform. Consistently with the welfare results, only 23% of the HL households and 27% of the HH households are in favor. Again, the age-30 households are the only age group that is against the reform. Majority of older households, as both sons' earnings and the likelihood of the father's death increase, again prefers the reform. The overall support for the reform is smaller than in the exogenous fertility case.

On the transition path, the aggregate capital stock falls immediately after the reform and reaches its steady state level after 20 years of transition. Because the labor supply decreases during the initial periods due to the time devoted to raising more children, the after-tax wage first rises and only then falls to its steady state level.

[19]Households where only sons are alive are better off regardless of their type or age. Of course, the households with only fathers alive experience a huge welfare loss from the uncompensated fully funded reform. Similarly to the exogenous fertility case, all newborn households into the FF steady state are better off than they were in the PAYG steady state (the cost of transition is between 0.9-1.2% depending on a household type).

5.3.4 Return on PAYG Social Security and Children

Another way to understand these welfare results is to examine the rate of return on social security in the PAYG system with endogenous fertility.[20] The return on social security for a newborn individual is computed from all contributions and benefits received during a person's lifespan. We find that the expected return is 2.7% for a low skill person and 2.4% for a high-skill person. Both these returns are lower than the after-tax return on savings.

The return on social security for complete households is shown in Table 6. This return depends on the composition of a household and mortality of its members. The rate of return on social security increases with the age of the father, who is collecting social security benefits for a longer time. Also, this rate is higher the lower the contributions made and the higher the benefits received. Hence the returns on PAYG are highest for the HL households and negative for the LH households no matter how long the father can collect the benefits (the ordering of returns across types is HL>LL>HH>LH). Obviously, households where only sons are alive have a negative return on social security.

INSERT TABLE 6 ABOUT HERE

When the life expectancy of the father is taken into account, only the HL household type has a social security return higher than the after-tax return on capital (7.9% vs. 4.7%). This confirms our findings above that HL households prefer the PAYG system the most while LH households with a negative return prefer the PAYG system the least.

The other part of this table displays the return on children for the PAYG and FF steady states with endogenous fertility. The return is calculated for a son making his fertility decision in period $j_N - 1$. The cost is the forgone earnings and consumption expenditures on children while the benefits consist of the future discounted earnings of these children in the next generation. Note that the son must take into account the future fertility decisions of his children and, more importantly, the realization of children's productivity skill after period T.

Because children start earning before their parent retires, the return on children in Table 6 is positive over more periods than that on social security benefits. Also, the differences between household types are smaller. The fully funded reform increases the average return on children by 1.1% (for households with high skill sons by 1.3%, for low skill sons by 0.9%).

Similarly to the return on social security, in both steady states the low skill sons (LL, HL) have higher return on children than the high skill sons. This is because their forgone earnings and child costs are lower while the future benefits can increase a lot if children

[20] See Fuster et al. (2003) for a definition and computation of the return. The returns on PAYG social security with exogenous fertility are similar to those listed here.

draw a high skill shock (and vice versa for high skill sons). In the FF steady state, low skill sons have an expected return 4.5% and the high skill sons 4.0% (in the PAYG steady state it is 3.6% and 2.7%, respectively).

5.4 Fully Funded Reform with Fertility Imposed from the PAYG Steady State

Finally, in order to ascertain whether these effects of the FF reform do not come solely from the fertility decisions already taken in the PAYG steady state and might not be, therefore, independent of the further fertility adjustments in the FF steady state, we compute a fully funded steady state on which we impose the fertility allocations chosen by households in the PAYG steady state with endogenous fertility. That is, while the households chose their fertility optimally under the PAYG system, they are not allowed to change their fertility later on when the fully funded reform takes place.

INSERT TABLE 7 ABOUT HERE

In Table 7 we see that the average number of children per son is slightly different from the PAYG economy with endogenous fertility. This is because we impose the same fertility allocations on households of the same individual state (savings and skills) but the distribution of these states differs in these two steady states due to different savings decisions under the PAYG and FF system.

With the imposed fertility allocations from the endogenous fertility PAYG steady state, the FF reform now increases the capital stock by 7.7% with all types of households contributing. Both steady states have a similar pattern of savings over the life cycle, namely, they allow the high skill households to transfer wealth across generations. The intervivos transfers and returns are also similar with respect to their sign and magnitude. Finally, welfare gains from transition are small but positive and the political support is only 56%.

The direction and magnitudes of these changes suggest that analyzing the fully funded reform using the fertility choices taken from the PAYG system might be as misleading as using the exogenous fertility model altogether.

5.5 Sensitivity Analysis and Other Results

In the remainder of this Section, we analyze the income and old-age support effects on fertility decisions. Then we follow Fuster et al. (2003) and present three additional cases of the PAYG and FF steady states with endogenous fertility that differ in lifetime uncertainty and/or skill differentials.

5.5.1 Income and Old-Age Support Effects

In the benchmark, endogenous fertility model, the general equilibrium effects are not so large as in the pure life-cycle, exogenous fertility models. In the FF reform, the lower capital stock and the elimination of the social security tax increase the equilibrium after-tax interest rate as well as wages. Consequently, the income of all household types is higher, in the range from 7% (the HL types) to 24% (the HH types). When children and the utility of children are complements in the utility of the parents, fertility should be increasing in income.

Given these predictions and responses by different household types, we analyze two effects present in the FF reform: First, the income effect from the elimination of the social security tax, and second, the role of old-age support provided by the PAYG benefits. In order to decompose these two effects, we simulate two counterfactual steady states in which the social security budget is obviously not balanced: In order to isolate the income effect, we eliminate the social security tax while preserving the PAYG benefits. In the second simulation, we keep the social security tax and eliminate the PAYG benefits to isolate the role of old-age support.

As these two effects represent the two pieces of which the FF reform consists of, each of them contributes to around 1/2 of the total fertility increase and capital decrease between the benchmark PAYG and FF steady states with endogenous fertility. However, each effect provides strong fertility incentives to different types of households: the income effect lowers the opportunity cost of children and is important for HH and H households (contributes to 60% and 80% of these households' fertility increase found in the benchmark reform, respectively). The old-age support effect is more important for LH households (represents 71% of the benchmark fertility increase; low skill fathers need more support from high skill sons) and L sons (80% of the benchmark fertility difference; these sons are poor and lose benefits).

This analysis suggests that the old-age support is important for households with a low skill father while the income effect from a reduced opportunity cost of children provides fertility incentives for the high skill households. Finally, the income and old-age effects have approximately the same impact on LL and HL households.

5.5.2 Case 1: Certain Lifetimes ($\psi = 1$)

In the first of our alternative calibrations of the benchmark model with endogenous fertility, we impose certain lifetimes ($\psi = 1$) for all household types.[21] At the cost of increased longevity (all agents now live till age 90), certain lifetimes eliminate the risk of losing the

[21] In all these cases the remaining parameters are the same as in the benchmark model.

old-age support because of sons' death. Note that there are only complete households.

Compared to the benchmark PAYG steady state, fertility in this PAYG steady state with certain lifetimes declines by 14.9% and assets fall by 8.6%. The removal of survival uncertainty reduces the need for the buffer stock of children (their future incomes) and savings.[22] Prolonging expected lifetimes reduces fertility and the population growth rate (1.06%). A smaller productive population contributes to lower output per capita.

Between the certain lifetimes steady states, the FF reform increases fertility by 14.1% while the capital stock further falls by 6.9%. The higher fertility response is driven by LL and LH households who increase fertility by more than 12% (other fertility changes are similar to those in the benchmark model). LH is the only type of household that increases its savings.

Naturally, the higher longevity increases the dependency ratio to more than 25% and the social security tax has to increase to 16.9% in order to finance the retirement benefits. The longer lives of low skill individuals substantially increase the fraction of LL types (61%).

Welfare gains have the same signs as in the benchmark reform (with HL households significantly more worse off). As there is a larger fraction of retired agents who lose their social security benefits, the FF reform in this case has the lowest political support (53%, still a majority).

5.5.3 Case 2: Equal Survival Probability ($\psi_H = \psi_L$)

In the second case, we keep uncertain lifetimes but impose the same survival probabilities for both high and low skill individuals ψ (computed as the weighted average of ψ_H and ψ_L). Households still differ in their skill composition.

When lifetimes are uncertain but the same for both skills, compared to the benchmark PAYG steady state fertility declines by 5.4% and the aggregate capital stock falls by 3.2%. The main reason is that the survival probabilities increase on average (i.e., for the most numerous low skill individuals) while skill uncertainty remains. The children of low skill parents are now more likely to survive and support the parents in their old age. This drives the fertility of low skill agents down in the PAYG system. On the other hand, high skill households now face a higher mortality risk and they do not dissave as much as in the first case with certain lifetimes.

Relative to this case's PAYG steady state, the FF reform increases fertility by 12% and reduces the capital stock by 3.1%. Responses by individual households are similar to the

[22] Kalemli-Ozcan (2002) stresses the role of survival uncertainty for the insurance strategy, or hoarding of children, where the actual number of children is greater than the optimal number of children for the parents.

previous case.

Compared to the benchmark specification, these two cases exhibit higher average survival probabilities. The fall in fertility in these two cases' PAYG steady states relative to the benchmark PAYG model suggests that children are used as insurance against survival uncertainty.

5.5.4 Case 3: Limited Heterogeneity ($\psi_H = \psi_L$, $\varepsilon_H = \varepsilon_L$)

The third case has no skill differences ($\varepsilon_H = \varepsilon_L$) and, therefore, the same but uncertain lifetimes ($\psi_H = \psi_L$). There are only three types of households: complete and incomplete with either father or sons alive. An equal survival uncertainty is the only risk in the economy.

With PAYG, fertility is close to that in the benchmark PAYG steady state (1.66 for complete households, 1.79 for lonely sons). As there is no need for the buffer stock except for life uncertainty and there are no high skill agents, the capital stock decreases by 22% relative to the benchmark PAYG steady state. Correspondingly, the output per capita declines as well. Relative to this case's PAYG steady state, the FF reform increases fertility by 7.8% while reducing capital by 2.7%. Welfare gains from the FF reform are large (for the average household) and the limited heterogeneity case obtains almost 100% support from all generations.

The large decline in savings in this case suggests again that in the PAYG system assets are primarily used to insure against a low future realization of skills among children.

6 Conclusions

Social security reform is one of the most important economic and political issues in the United States and other developed countries. This paper analyzes the social security reform in a general equilibrium model with altruistic dynasties and endogenous fertility. The main actors of this model are the high skill agents. In the PAYG system, these agents save relatively more in assets than in children. Consequently, models with exogenous fertility may impose a too high fertility on these households and underestimate the aggregate capital stock in the PAYG steady state. We show that the magnitude of this error might be greater than the effect of the fully funded reform itself.

Further, the high skill households' responses to FF reform are much stronger than those of the low skill households. As high skill agents switch to investing in the old-age support from children rather than in assets, fertility increases while the capital stock falls. Thus the FF reform with endogenous fertility leads to opposite aggregate outcomes than the same reform with exogenous fertility. The fertility margin during the FF reform is important.

We show that analyzing the fully funded reform with imposed fertility choices from the PAYG system might be as misleading as using the exogenous fertility model altogether.

The FF reform reduces not only fertility but also wealth differences across different household types. This is because the retirement benefits of the PAYG system allow the wealthy households to bequest most of their assets to future generations. The fully funded reform reduces inequality by eliminating this wealth persistence, providing instead incentives for investment in children and life-cycle savings. Together, increased fertility and intervivos transfers from children are able to partially replace the insurance mechanisms provided by the PAYG social security system. It seems that children are used as insurance against survival uncertainty while assets against the risk associated with uncertain skills in future generations.

The FF reform is supported by a large majority of population as the dynastic framework with two-sided altruism allows for compensation of the transition costs within families and across generations. Importantly, poorer households experience a welfare gain while the rich households tend to lose (mostly because they need to increase the number of children during the initial periods of the transition).

This life-cycle dynastic model with endogenous fertility is open to other extensions such as the incorporation of endogenous labor, more detailed and optimal fiscal and social security reforms or government policies allowing for child and maternity support.

Appendix. Life-Cycle Intervivos Transfers

This Appendix presents the computation of intervivos transfers. Recall that all adult members of a household have the same objective function, they pool their resources and solve a joint maximization problem. As in Fuster, Imrohoroglu, and Imrohoroglu (2003), we assume that the father owns all assets of the household in period $j = 1$. Denote assets owned by the father at period j as $a_f(j)$ and those owned by each son as $a_s(j)$. The laws of motion of these asset holdings follow the individual budget constraints,

$$\begin{aligned}(1+g)a_f(j+1) &= [1+r(1-\tau_k)]a_f(j) + e_f(j) + \xi(j) - s \cdot x(j) - (1+\tau_c)(c(j) + \gamma^g(j)),\\ (1+g)a_s(j+1) &= [1+r(1-\tau_k)]a_s(j) + e_s(j) + \xi(j) + x(j) - (1+\tau_c)(c(j) + \gamma^g(j)),\end{aligned}$$

where $e_f(j)$ and $e_s(j)$ are after-tax earnings of the father and the son, $\xi(j)$ is the accidental bequest transfer from the government, $c(j)$ is the equal consumption of each adult member and $\gamma^g(j)$ are expenditures on children per household member. The intervivos transfer from the father to each son in period j is denoted by $x(j)$. A negative transfer is a support from each son to the father.

A person living in the household needs a transfer if his individual consumption is larger than his total income in a particular period. For example, if a son's individual budget constraint implies that

$$[1+r(1-\tau_k)]a_s(j) + e_s(j) + \xi(j) - (1+\tau_c)\gamma^g(j) < (1+\tau_c)c(j),$$

then he needs to receive a transfer of

$$x(j) = (1+\tau_c)(c(j) + \gamma^g(j)) - [1+r(1-\tau_k)]a_s(j) - e_s(j) - \xi(j).$$

Naturally, the son's assets in the next period will be set to zero, $a_s(j+1) = 0$, and all assets of the household will be owned by the father, $a_f(j+1) = a(j+1)$.

If total incomes of the father and of each son are sufficient to finance their consumption, then the transfer is zero, $x(j) = 0$, and the members distribute the next period assets according to their relative income contribution.

Compared to Fuster et al. (2003) and in Fuster et al. (2007), our model with endogenous fertility requires an assumption which members of the household bear the cost of raising the children $\gamma^g(j)$. As the household shares all incomes and expenditures, we assume that the child cost is also shared by all adult members.

References

Alvarez, F. (1999). Social mobility: The barro-becker children meet the laitner-loury dynasties. *Review of Economic Dynamics* (2), 65–103.

Auerbach, A. J. and L. J. Kotlikoff (1987). *Dynamic Fiscal Policy*. New York, N. Y.: Cambridge University Press.

Barro, R. J. and G. Becker (1989). Fertility choice in a model of economic growth. *Econometrica 57*(2), 481–501.

Boldrin, M., M. De Nardi, and L. E. Jones (2005). Fertility and social security. *NBER Working Paper No. 11146*.

Boldrin, M. and L. Jones (2002). Mortality, fertility and savings decisions. *Review of Economic Dynamics 5*, 775–814.

Caldwell, J. C. (1982). *Theory of Fertility Decline*. New York: Academic Press.

Conesa, J. C. and C. Garriga (2008). Optimal fiscal policy in the design of social security reforms. *International Economic Review 49*, 291–318.

Conesa, J. C. and D. Krueger (1999). Social security reform with heterogeneous agents. *Review of Economic Dynamics 2*(4), 757–795.

De Nardi, M., S. Imrohoroglu, and T. J. Sargent (1999). Projected u.s. demographics and social security. *Review of Economic Dynamics 2*(3), 576–615.

Deaton, A. S. and J. Muellbauer (1986). On measuring child costs: With applications to poor countries. *Journal of Political Economy 94*(4), 720–744.

Elo, I. T. and S. H. Preston (1996). Educational differentials in mortality: United states, 19791985. *Social Science and Medicine 42*, 47–57.

Fuster, L. (1999). Is altruism important for understanding the long-run effects of social security? *Review of Economic Dynamics 2*(3), 616–637.

Fuster, L., A. Imrohoroglu, and S. Imrohoroglu (2003). A welfare analysis of social security in a dynastic framework. *International Economic Review 44*(4), 1247–1274.

Fuster, L., A. Imrohoroglu, and S. Imrohoroglu (2007). Elimination of social security in a dynastic framework. *International Economic Review 74*(1), 113–145.

Gale, W. G. and J. K. Scholz (1994). Intergenerational transfers and the accumulation of wealth. *Journal of Economic Perspectives 8*, 145–161.

Golosov, M., L. E. Jones, and M. Tertilt (2007). Efficiency with endogenous population growth. *Econometrica 75*(4), 1039–1071.

Imrohoroglu, A., S. Imrohoroglu, and D. Joines (1999). Social security in an overlapping generations economy with land. *Review of Economic Dynamics 2*, 638–665.

Jones, L. E. and A. Schoonbroodt (2007). Complements versus substitutes and trends in fertility choice in dynastic models. *University of Minnesota Working Paper*.

Jones, L. E., A. Schoonbroodt, and M. Tertilt (2008). Fertility theories: Can they explain the negative fertility-income relationship? *NBER Working Paper 133631*.

Jones, L. E. and M. Tertilt (2006). An economic history of fertility in the u.s.: 1826-1960. *NBER Working Paper No.12796*.

Kalemli-Ozcan, S. (2002). A stochastic model of mortality, fertility, and human capital investment. *Working Paper*.

Lucas, Jr., R. E. (2002). The industrial revolution: Past and future. In *Lectures on Economic Growth*, pp. 109–190. Cambridge: Harvard University Press.

Nishimura, K. and J. Zhang (1992). Pay-as-you-go public pension with endogenous fertility. *Journal of Public Economics 48*, 239–258.

Rothbarth, E. (1943). Note on a method of determining equivalent income for families of different composition. In C. Madge (Ed.), *War-Time Pattern of Saving and Spending*, Cambridge. Cambridge University Press.

SHARE (2005). *Health, Aging and Retirement in Europe: First Results from the Survey of Health, Aging and Retirement in Europe*. Mannheim Research Institute for the Economics of Aging (MEA), Germany.

Storesletten, K., C. Telmer, and A. Yaron (1999). The risk sharing implications of alternative social security arrangements. *Carnegie-Rochester Conference Series on Public Policy 50*, 213–260.

Zimmerman, D. (1992). Regression toward mediocrity in economic stature. *American Economic Review 82*, 409–429.

	Parameters	
	Population	
j_{2T}	$= 14$	Maximum lifetime (90 years)
j_R	$= 10$	Retirement age (65 years)
j_N	$= 4$	Fertility age (35 years)
$\bar{n}_{USA}$	$= 0.012$	Population growth rate U.S.
ψ		Survival probabilities
	Utility	
β	$= 0.988$	Annual discount factor
σ	$= 0.95$	Relative risk aversion
η	$= 0.055$	Altruism
	Production	
g	$= 0.014$	Annual technology growth
δ	$= 0.044$	Annual depreciation rate
α	$= 0.34$	Capital share
$\pi_{LL} = 0.83$	$\pi_{HH} = 0.57$	Transition matrix for skills
e		Earnings profiles
	Fiscal Policy	
τ_k	$= 0.35$	Capital income tax rate
τ_c	$= 0.055$	Consumption tax rate
G	$= 0.225$	Government purchases (% GDP)

Table 1: Parameters

Cost of Children: Working Time		
Age of	Fraction of Combined Hours (γ_w)	
(Younger) Child	Weekly	Annual
0-4	0.0441	0.0755
5-9	0.0392	0.0662
10-14	0.0392	0.0662
15-19	0.0392	0.0662

Cost of Children: Expenditures		
Age of (Younger) Child	Annual Expenditure	Fraction of Income (γ_g)
One-Child Household		
0-4	\$10,354 = \$8,350 · 1.24	0.22
5-9	10,540 = 8,500 · 1.24	0.22
10-14	11,226 = 9,054 · 1.24	0.24
15-19	11,408 = 9,200 · 1.24	0.25
Two-Child Household		
0-4	\$17,690 = \$8,350 + \$9,340	0.36
5-9	17,840 = 8,500 + 9,340	0.37
10-14	18,394 = 9,054 + 9,340	0.38
15-19	18,540 = 9,200 + 9,340	0.39
n-Child Household ($n > 2$)		($n = 3$)
0-4	(\$8,350 + \$9,340 + (n-2)· \$9,200) · 0.77	0.43
5-9	(8,500 + 9,340 + (n-2)· 9,200) · 0.77	0.43
10-14	(9,054 + 9,340 + (n-2)· 9,200) · 0.77	0.44
15-19	(9,200 + 9,340 + (n-2)· 9,200) · 0.77	0.45

Sources: Working time: *Report on the American Workforce.* 1999. U.S. Department of Labor. Table 3-6. Expenditures: Table 9 in *Expenditures on Children by Families. 1998 Annual Report*, United States Department of Agriculture. Miscellaneous Publication Number 1528.

Table 2: Cost of Children

PAYG Steady States with Exogenous and Endogenous Fertility

Fertility	τ_{ss}	K	N	Y	C	K/Y	$r(1-\tau_k)$	$w(1-\tau_{ss}-\tau_l)$
Exogenous	0.115	0.613	1.781	1.239	0.521	2.472	0.056	0.332
Endogenous	0.115	0.757	1.785	1.333	0.549	2.839	0.047	0.357
(%)		+23.4	+0.2	+7.6	+5.4			+10.8

	Both f and s Alive				Only s Alive			
Fertility	LL	HL	LH	HH	L	H	Average	Growth
Exogenous	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.2%
Endogenous	1.83	1.56	1.65	1.11	1.85	1.61	1.67	1.2%
(%)	+9.6	-6.6	-1.2	-33.5	+10.8	-3.5		
Assets (%)	+22.9	+29.2	+16.3	+33.1	+22.8	+35.0	+23.4	

Table 3: PAYG Steady States with Exogenous and Endogenous Fertility

PAYG and FF Reform with Exogenous Fertility								
Steady State	τ_{ss}	K	N	Y	C	K/Y	$r(1-\tau_k)$	$w(1-\tau_{ss}-\tau_l)$
PAYG	0.115	0.613	1.781	1.239	0.521	2.472	0.056	0.332
FF	0.00	0.656	1.781	1.268	0.528	2.586	0.053	0.395
(%)		+7.0	0.0	+2.3	+1.1			+18.9
	Both f and s Alive				Only s Alive			
	LL	HL	LH	HH	L	H	Average	Growth
Fertility	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.2%
Assets (%)	+11.5	+2.4	+9.3	+4.2	+8.4	+6.8	+7.0	
Transition Analysis (in %)								
Welfare Gain	+1.8	+1.2	+2.2	+1.6	+3.1	+2.8	+2.1	
Political Supp.	100	83	100	100	100	100	96	
By Age:	20	25	30	35	40	45	50	
Welfare Gain	+2.6	+1.8	+1.0	+1.7	+2.1	+2.4	+2.7	
Political Supp.	100	100	88	100	100	100	100	

Table 4: Steady States and Transition with Exogenous Fertility.

PAYG and FF Reform with Endogenous Fertility								
Steady State	τ_{ss}	K	L	Y	C	K/Y	$r(1-\tau_k)$	$w(1-\tau_{ss}-\tau_l)$
PAYG	0.115	0.757	1.785	1.333	0.549	2.839	0.047	0.357
FF	0.00	0.694	1.889	1.344	0.536	2.582	0.053	0.394
(%)		-8.3	+5.8	+0.8	-2.4			+10.1
	Both f and s Alive				Only s Alive			
	LL	HL	LH	HH	L	H	Average	Growth
Fertility PAYG	1.83	1.56	1.65	1.11	1.85	1.61	1.67	1.2%
Fertility FF	1.88	1.89	1.78	1.73	1.80	1.76	1.84	1.5%
(%)	+2.7	+21.2	+7.9	+57.3	-2.7	+9.3	+10.2	
Assets (%)	+0.5	-22.2	+3.5	-23.7	-4.1	-19.4	-8.3	
Transition Analysis (in %)								
Welfare Gain	+0.5	-0.5	+0.9	-0.3	+1.4	+1.1	+0.5	
Political Supp.	79	23	94	27	100	100	73	
By Age:	20	25	30	35	40	45	50	
Welfare Gain	+1.2	+0.5	-0.4	+0.1	+0.3	+0.5	+0.6	
Political Supp.	92	76	20	63	72	87	92	

Table 5: Steady States and Transition with Endogenous Fertility

Return on PAYG and Children Endogenous Fertility				
f's Age	Both f and s Alive			
at Death	LL	HL	LH	HH
Return on PAYG				
65	<0	<0	<0	<0
70	<0	<0	<0	<0
75	<0	6.5	<0	<0
80	4.1	8.5	<0	3.9
85	6.2	9.4	<0	5.9
E	3.2	7.9	<0	3.1
Return on Children in the PAYG Steady State				
65	1.5	1.1	<0	<0
70	3.0	2.7	0.9	1.7
75	4.0	3.7	2.2	2.9
80	4.7	4.4	3.0	3.6
85	5.1	4.8	3.5	4.1
E	3.7	3.4	2.4	3.0
Return on Children in the Fully Funded Steady State				
65	2.7	2.1	1.5	1.1
70	4.1	3.6	3.1	2.7
75	5.0	4.5	4.1	3.8
80	5.6	5.1	4.7	4.4
85	6.0	5.5	5.2	4.8
E	4.6	4.2	4.1	3.8

Notes: Conditional on father's survival, complete households.
Expected returns of newborns on social security in the PAYG steady state: $r_{SS}(L)$=2.7, $r_{SS}(H)$=2.4.
Expected returns on children in the PAYG steady state:
r_N=3.1, $r_N(Lson)$=3.6, $r_N(Hson)$=2.7.
Expected returns on children in the FF steady state:
r_N=4.2, $r_N(Lson)$=4.5, $r_N(Hson)$=4.0.

Table 6: Return on PAYG and Children with Endogenous Fertility.

FF Reform with Imposed Fertility from the PAYG Steady State								
Steady State	τ_{ss}	K	L	Y	C	K/Y	$r(1-\tau_k)$	$w(1-\tau_{ss}-\tau_l)$
PAYG	0.115	0.757	1.785	1.333	0.549	2.839	0.047	0.357
FF	0.0	0.815	1.810	1.379	0.562	2.955	0.044	0.424
(%)		+7.7	+1.4	+3.5	+2.3			+18.8
	Both f and s Alive				Only s Alive			
	LL	HL	LH	HH	L	H	Average	Growth
Fertility PAYG	1.83	1.56	1.65	1.11	1.85	1.61	1.67	1.2
Fertility FF	1.82	1.53	1.73	1.16	1.90	1.64	1.67	1.2
(%)	-0.1	-1.9	+4.8	+4.5	+2.7	+1.9	+0.6	
Assets (%)	+9.9	+6.3	+9.8	+5.2	+12.4	+2.6	+7.7	
Transition Analysis (in %)								
Welfare Gain	+0.3	-1.2	+0.5	-1.1	+1.2	+0.7	+0.1	
Political Supp.	64	21	68	25	100	100	56	
By Age:	20	25	30	35	40	45	50	
Welfare Gain	+0.6	+0.0	-0.7	-0.3	+0.0	+0.2	+0.3	
Political Supp.	72	57	18	36	52	65	71	

Table 7: Steady State and Transition with Fertility Imposed from the PAYG Steady State.

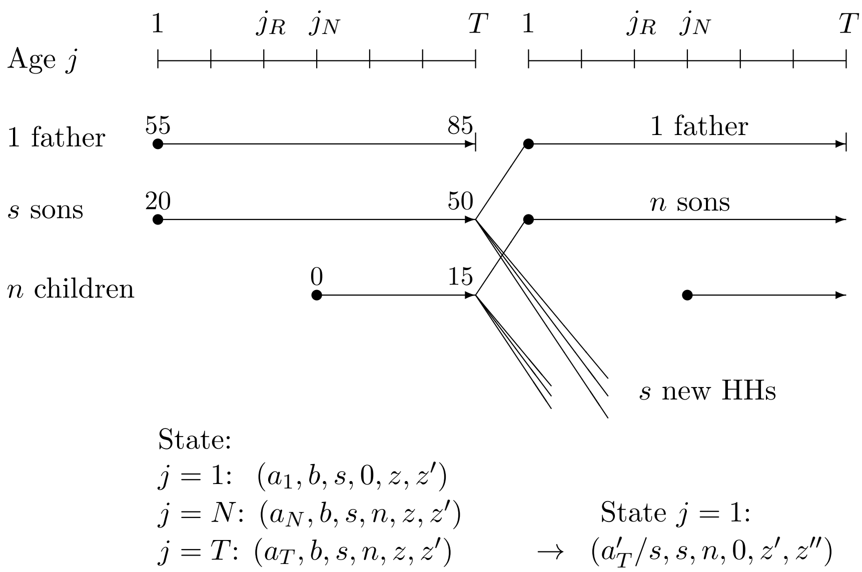

Figure 1: Timeline for Households

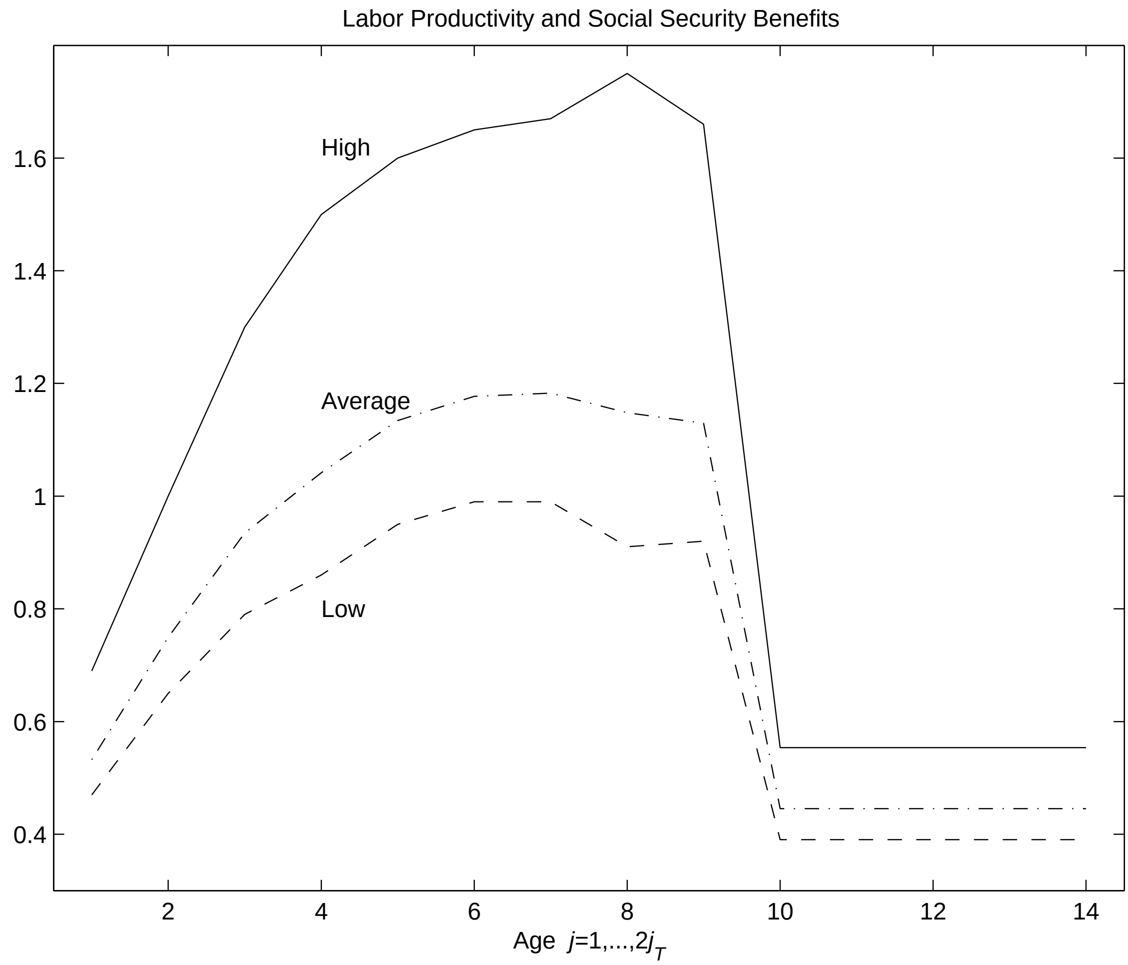

Figure 2: Age Profiles of Labor Productivity and Social Security Benefits.

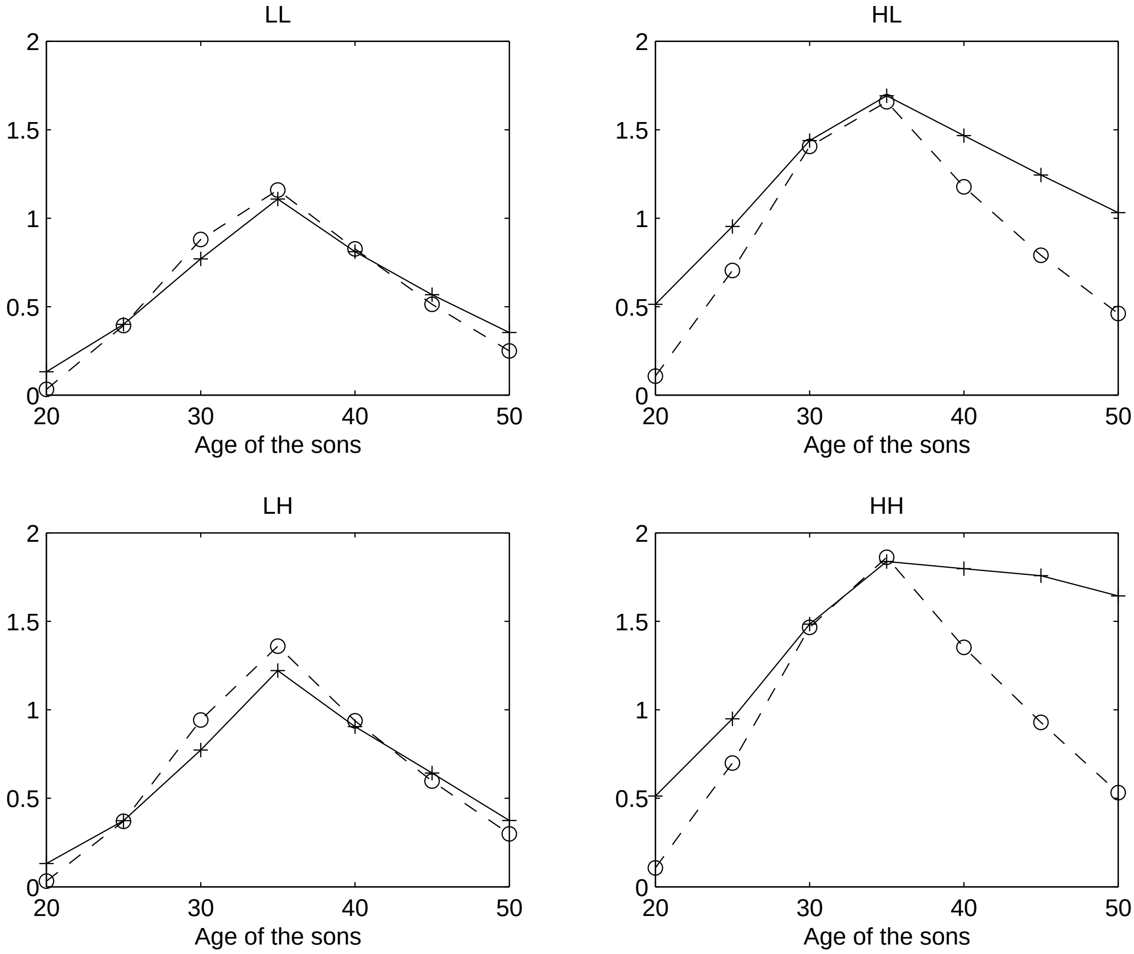

Figure 3: Average Wealth in Complete Households. PAYG: Full line (+). FF: Dashed line (○). Endogenous Fertility.

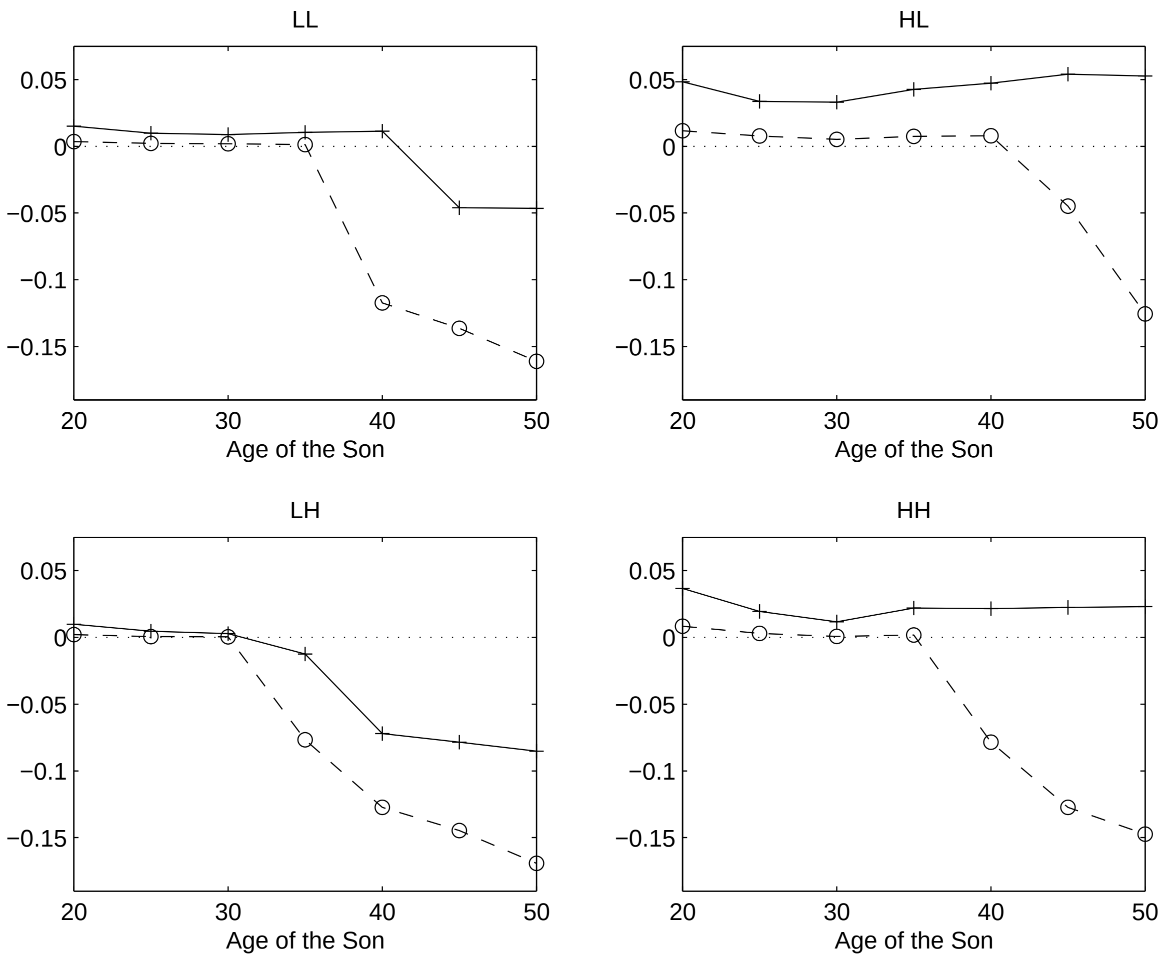

Figure 4: Average Intervivos Transfer from Father to One Son in Complete Households. PAYG: Full line (+). FF: Dashed line (○). Endogenous Fertility.