Financial Constraints and Entrepreneurial Investment

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Abstract

This paper studies the interdependence between the investment and saving decisions of entrepreneurial households in a dynamic, general equilibrium model with heterogeneous agents and occupational choice. Because of limited commitment, whether an agent becomes an entrepreneur depends on entrepreneurial skills and accumulated assets needed to finance a business project with uncertain returns. The simulated economy replicates the U.S. data on distribution of wealth and income, and the shares of wealth and income of each occupation. The dominant incentive behind the high levels of savings of business households is the desire to relax the wealth constraint in financing the entrepreneurial projects and to expand the firms to their optimal size. Finally, as firms endogenously grow over time, most entrepreneurs enter despite lower initial earnings than they would receive in paid employment.

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1 Introduction

Gentry and Hubbard (2000) report three main findings about entrepreneurial saving decisions and their role in household wealth accumulation. First, entrepreneurial households own a substantial share of household wealth and income, and this share increases throughout the wealth distribution and the income distribution. Second, the portfolios of wealthy households are very undiversified with the bulk of assets held in active business. Third, wealth-income ratios and saving rates are higher for entrepreneurial households even controlling for age and other demographic variables.

In this paper I study the interdependence between the investment and saving decisions of entrepreneurial households. I build a dynamic, general equilibrium model with heterogeneous agents in which each agent faces the following option: to work as a worker or to become an entrepreneur. The occupational heterogeneity is important as workers lend their assets to entrepreneurs who can use them more productively.[1] Each agent compares the expected value he or she would obtain from a paid job to the expected value of the profits accruing from running a firm. A worker receives a wage while an entrepreneur establishes a firm with capital investment, employs other agents as workers, and realizes profit from a decreasing returns to scale production technology. The business investment plans influence the saving decisions in three ways: First, entrepreneurs save more to insure against business risk. Second, potential entrepreneurs increase their total savings in anticipation of future investment needs. Third, successful entrepreneurs reinvest business earnings in further business expansion.

This interdependence is a result of financial constraints. In theory, the investment and saving decisions are independent under the assumption of perfect capital markets. As in Banerjee and Newman (1993), the financial constraint is motivated by limited commitment.[2] Because of capital market imperfections enforcement of loan contracts is imperfect, and agents have an opportunity to renege. In my model, entrepreneurs borrow only limited amounts secured by a collateral. The collateral (accumulated

[1] See Kiyotaki and Moore (2001) for a similar setup in which a fraction of agents has an investment opportunity.

[2] Another possible motivation is the moral hazard problem in Aghion and Bolton (1997).

assets) guarantees not only the repayment of the loan but also a positive consumption in the case of a project's failure. As the financial constraint is endogenously related to a borrower's wealth, entrepreneurship is positively correlated with wealth.

The low-wealth agents may be constrained in investment and some may be unable to borrow to finance their business projects in spite of having a very good entrepreneurial idea. In fact, if a risk-averse entrepreneur must invest disproportionately into one irreversible project with uncertain returns, he may forgo the plan and work for a wage. In sum, the level of accumulated individual wealth together with an idiosyncratic entrepreneurial skill are the decisive forces behind each agent's ability to finance and actually undertake an entrepreneurial project. Firms grow endogenously over time as the financial constraint is alleviated from reinvested business profits.

The simulated economy delivers most of the distributional features of the U.S. economy. In particular, its steady state replicates the Gini coefficient of inequality for both wealth and income as well as the distribution of assets and income in the top percentiles.[3] For entrepreneurs and workers, I match the share of wealth owned and the share of income received by each occupation and the main wealth-income ratios.

I find that the dominant factor behind the high levels of savings of business households is the incentive to accumulate assets in order to relax the wealth constraint in financing the entrepreneurial projects, and therefore, to operate their businesses at the optimal size. In other words, I confirm Klein (1960) claim that "[o]f primary importance is the need and desire of entrepreneurs to reinvest their unspent business earnings in further business expansion." As in Hamilton (2000) empirical study, most entrepreneurs enter business despite lower initial earnings than they would receive in paid employment. In my model, the credit-constrained entrants not only have lower income but they also sacrifice current consumption for investment in the future growth of their businesses.

These results suggest that occupational choice and entrepreneurial decisions subject to credit constraints have important implications for explaining the high concentration of wealth observed in the U.S. economy. The modelling strategy departs from

[3] The standard models with uninsurable shocks to labor earnings and borrowing constraints (Huggett (1996) or Aiyagari (1994)) severely under-predict the degree of wealth inequality.

the neoclassical theory of investment with a representative agent where, according to the Modigliani-Miller theorem, the firm's optimal capital stock is independent of financial factors. Instead, it follows the Lucas (1978) study of business firms' size and the Lucas (1990) and Fuerst (1992) models with *ex ante* uncertainty related to investment decisions. Compared to more recent models of occupational choice (Quadrini (1999a), De Nardi and Cagetti (2001), Erosa (2001), Jeong and Townsend (2001), or Gomes, Greenwood, and Rebelo (2001)) this model preserves the simplicity of the dynamic, general equilibrium neoclassical models.

The paper is organized as follows. The next Section surveys the empirical evidence on entrepreneurial activity and distribution of resources in the U.S. economy. Section 3 develops the main model and defines a stationary recursive equilibrium. Characterization of occupational and entrepreneurial choices are described in Section 4. Parameters and results of a numerical simulation are presented in Section 5. Section 6 concludes.

2 Entrepreneurial Activity in the United States

I follow Gentry and Hubbard (2000) and define entrepreneur as someone who combines upfront business investment with entrepreneurial skill to obtain the chance of earning economic profits. According to the Survey of Consumer Finances (SCF, 1989), 8.7% households report active business assets greater than $5,000 (9.5% report business assets greater than $1,000). Similarly, in the Panel Study of Income Dynamics (PSID 1994) 10.4% of families own a business or have a financial interest in some business enterprise.

1) **Wealth and Income Inequality** In the 1990s, the Gini coefficient for family wealth have been between 0.78 and 0.84, depending on the year and survey (PSID and SCF, respectively). The Gini coefficient for family income is 0.45 in the PSID and declines to 0.34-0.36 after government transfers. Wealth is much more concentrated than income also within each occupational group (0.69 for entrepreneurs and 0.73 for workers).

In the PSID, the top 1 percent of families owns around 29% of the total household wealth and around 8% of the total income. The top 5 percent owns already 50% of the wealth and receives 20% of the income. Finally, the top docile owns more than 60% of the wealth and receives more than 32% of the income. These numbers are usually higher by 7-10 percent in the SCF surveys.

2) **Entrepreneurial Wealth and Savings** The percentage of business families increase in higher wealth classes: Quadrini (1999a) documents that about half of the families in the top 5% are business families. The concentration of wealth among business families is not purely explained by the concentration of income. Quadrini (1999b) and Gentry and Hubbard (2000) report that entrepreneurs are wealthy because they not only earn more income but also save relatively more than workers. Entrepreneurs, being such a small fraction of the population, receive 22% of the total income and own 40% of the total wealth. The ratio of wealth to income is about twice as large for business families (6.77 versus 2.94).

3) **Entrepreneurial Projects** Entrepreneurial portfolios are very undiversified. Gentry and Hubbard (2000) find that active businesses account for 42% of entrepreneurs' assets (even in the top wealth classes) and relative to non-entrepreneurs, entrepreneurs hold less of their wealth in liquid assets. In the Characteristics of Business Owners (CBO, 1992) more than 75% of entrepreneurs own only one business. The turnover of business families is substantial. Entrepreneurial income is more volatile than the labor income of workers. Heaton and Lucas (2000) find that the median standard deviation of the growth rate of nonfarm proprietary income is 64% annually, and the median standard deviation of the growth rate of real wage income is only 35% annually.

4) **Financial Constraints** Available evidence suggests that entrepreneurs are constrained by their wealth. Based on the National Longitudinal Survey, Evans and Leighton (1989) find that men with greater assets are more likely to become self-employed all else being equal. They estimate in their model that entrepreneurs can borrow up to 50% of their current assets.[4] The Federal Re-

[4] In an important field study Paulson and Townsend (2002) find two-thirds of Thai business

serve Survey of Terms of Business Lending reveals that small loans are more often secured by collateral.[5]

Small firms pay fewer dividends, take on more debt, and invest more. In terms of the aggregate value of small firm debt, almost 90% of credit comes from traditional sources, mostly from lines of credit and loans. Almost half of the entrepreneurs use their own or family's savings and smaller entrepreneurs also use physical assets. Fazzari, Hubbard, and Petersen (1988) report that internal finance in the form of retained earnings generates the majority of net funds for firms of all size categories: the average retention ratio is largest for small firms (80%) and lowest for the largest firms (50%). Finally, Eisner (1978) finds that the timing of investment in small firms is more sensitive to profits than it is in large firms.

5) Occupational Choice Hamilton (2000) finds evidence that most entrepreneurs enter and persist in business despite the fact that they have lower initial earnings in paid employment, with a median earnings differential of 35 percent.

The model in the next Section is supposed to replicate this list of data on entrepreneurial activity in the United States. Motivated by the above empirical regularities, agents will be identified by their accumulated level of assets and entrepreneurial ability. In the presence of financial constraints, occupational choice and entrepreneurial decisions will be functions of this individual state and equilibrium prices.

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households financially constrained, mostly due to limited commitment problem faced by the poor entrepreneurs.

[5] In 2000, of all commercial and industrial loans in the United States, 83% required collateral for loans smaller than \$99,000, 74% for loans smaller than \$1 million, 46.9% for loans smaller than \$10 million, and only 31.7% for loans greater than \$10 million.

3 The Model

The economy is populated by a continuum of infinitely lived agents on a unit interval. Each agent has preferences over consumption given by a utility function

$$E\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right],$$

where $\beta \in (0,1)$ and $u : \mathbb{R}_+ \to \mathbb{R}$ is bounded, strictly increasing, strictly concave, and a twice differentiable continuous function that satisfies the Inada conditions.

In the beginning of a period each agent is identified by a level of accumulated assets $a \in A = [0,\infty]$ and by an ability shock $z \in Z = [\underline{z}, \overline{z}]$. The ability shock is carried from the previous period and is interpreted as a signal for an effective ability shock $z' \in Z$ that is realized later in the current period.

Production of the consumption good occurs in a large number of firms. Each firm is owned and managed by one entrepreneur who rents k units of capital and hires n workers in competitive factor markets. An entrepreneur with effective ability z' produces output $y = z'f(k,n) = z'(k^{\alpha}n^{1-\alpha})^{\theta}$, where $\alpha \in (0,1)$ and $\theta < 1$. The production function exhibits decreasing return to managerial control as in Lucas (1978) with the expected profit share $1-\theta$. These assumptions preclude pyramidal managerial structure and ensure that even the best managers run projects of a finite size. Capital used in production depreciates at a rate $\delta \in (0,1)$.

There is a competitive labor market where entrepreneurs hire workers at an equilibrium wage, w, and a competitive banking sector with which all agents deposit their accumulated assets. In the whole paper I assume that the banks provide only one period loan contracts with limited enforceability. Therefore, the banks require that the borrowers hold sufficient amount of assets (collateral) to be able to repay the loan even if the business project fails.[6] As there is no cost of intermediation and no default, the banks pay the same equilibrium interest rate, r, both on deposits and one period loans to entrepreneurs. At these equilibrium prices, the agents who find it more valuable to be workers provide their savings and labor to those agents that

[6]I also assume that a project's failure does not lead to a full depreciation of the capital used in production, or in other words, that the capital is not project-specific.

are currently more productive as entrepreneurs. Because of the decreasing returns to scale technology, some of the very wealthy entrepreneurs are net lenders too.

Given the individual state (a, z) at the beginning of the period, each agent decides whether to work as a worker or whether to become an entrepreneur. If the agent becomes an entrepreneur, he or she must decide how much inputs (k, n) to contract *before* the effective ability shock z' is drawn from a first-order Markov process, $Q(z, z')$. Depending on the effective ability z', profits might be positive or negative. In particular, if the realized z' turns out to be very low (say zero), the profit is negative since the entrepreneur must repay the loan, $(r + \delta)k$, and the contracted workers, wn. If the agent becomes a worker, he or she draws the effective ability shock z' from a fixed distribution ψ and receives labor income $z'wl$ where $l = 1$ is the normalized working time. I assume that each firm hires workers of the same average effective ability $\int z' \psi(dz')$.

The expected forgone wage serves as an endogenous entry cost for entrepreneurs. Therefore, it will not be optimal to establish a firm of a very small size with expected profits much lower than the expected wage from employment. Entrepreneurial risk is another cost of establishing a business. Each entrepreneur faces a positive probability of realizing a very low effective ability shock and having a negative profit he or she must expense from the accumulated assets. With the entrepreneurial project committed before the effective ability shock is realized, agents with a low level of assets will not take any or at least large entrepreneurial projects even if they have a very high entrepreneurial ability.[7]

The Markov structure of shocks to entrepreneurial ability reflects the learning aspect of entrepreneurial success as documented in Quadrini (1999a). I assume that Q is monotone and satisfies the Feller property. I abstract from a fixed cost associated with operating a business modelled in Hopenhayn and Rogerson (1993), among others. Instead, I follow Veracierto (2001) and assume that $\underline{z}$ is such a low

[7] Note that if the agents could choose their occupation after they observe the effective ability shock, all agents with high effective ability would become entrepreneurs and choose the profit-maximizing size of the firm regardless of their wealth. Since the purpose of this study is the relationship between entrepreneurial savings and investment, I model the occupational choice as an *ex ante* commitment of resources with uncertain return.

entrepreneurial skill with $Q(\{\underline{z}\},\{\underline{z}\}) = 1$ that even the wealthiest agents with this signal always prefer to work for wage. Second, to guarantee the exit of entrepreneurs, I assume that $Q(z,\{\underline{z}\}) > 0$ for all $z \in Z$. The occupational choice as a function of assets and entrepreneurial ability is summarized in Figure 1. The values of the shocks and the implications of such a structure of the transition matrix will be treated in detail in the following Section.[8]

After the labor income or profit is realized, both workers and entrepreneurs decide on how much to consume and on the amount of assets invested to the next period. Each agent carries his or her effective ability shock z' drawn after the occupational choice as the next period signal for future effective ability shocks. The decision of an agent identified by the asset level and ability signal, (a, z), can be formalized by the following recursive optimization problem

$$v(a,z) = \max\left\{\int v^W(a,z')\,\psi(dz'), \max_{k,n}\int v^E(a,z')\,Q(z,dz')\right\}, \tag{1}$$

where the value for a worker and an entrepreneur, $I \in \{W, E\}$, respectively, equals

$$v^I(a,z') = \max_{c,a'}\{u(c) + \beta v(a',z')\}, \tag{2}$$

subject to the following occupation-specific constraints. The worker's budget constraint is

$$c + a' \leq (1+r)a + z'wl. \tag{3}$$

The entrepreneur's budget constraint, given the choice of capital investment, k, and hired labor, n, equals

$$c + a' \leq (1+r)a + \pi(k,n,z'), \tag{4}$$

where $\pi(k, n, z')$ is the current period's profit,

$$\pi(k,n,z') = z'f(k,n) - (r+\delta)k - wn. \tag{5}$$

Finally,

$$a \in A \text{ with } \underline{a} = 0, \text{ and } k,n \geq 0, l = 1. \tag{6}$$

[8]Observe that if both types of agents did not draw idiosyncratic ability shocks, over time all agents would be of the same occupation and the equilibrium would not exist.

Note that it is optimal for an agent who decides to be a worker not to take any loan.

The specification of the Inada-type utility function together with the uncertainty in entrepreneurial profits imply that agents with a low level of accumulated assets may be constrained with respect to the size of the entrepreneurial project. In particular, the total entrepreneurial income, $(1+r)a+\pi(k,n,z')$, must guarantee a nonnegative consumption for all possible realizations of profit,

$$(r+\delta)k + wn < (1+r)a + z' f(k,n) \ \text{ for all } \ z' \in Z. \tag{7}$$

Since in each period $Q(z,\{\underline{z}\}) > 0$ for all $z \in Z$, this financing constraint must be satisfied for the lowest effective ability shock $\underline{z}$, which I set to zero. As the financing constraint depends only on the asset level and not on the signal of the effective entrepreneurial ability z, poor agents with good entrepreneurial ideas may not be able to establish a firm or the firm size may be smaller than it would have been without the financing constraint.[9]

3.1 Stationary recursive equilibrium

The concept of stationary recursive equilibrium requires that assets supplied by all agents equal the amount of capital demanded by the entrepreneurs, that labor supply by workers equals the labor hired by entrepreneurs, and that all allocations be feasible for a time invariant distribution of agents over their types.

The policy function for the next-period assets $a'(a,z)$ and the laws of motion for the ability shock process generate a law of motion for the distribution of agents over their individual states,

$$\lambda'(A',Z') = \int_{\{(a,z'):a'(a,z')\in A'\}} \Delta(z,dz')\, \lambda(da \times dz)$$

for all A' and Z', where $\Delta(z,dz') \equiv Q(z,dz')_{|E} + \psi(dz')_{|W}$ selects the law of motion for entrepreneurs' and workers' ability shocks. The probability measure λ describes

[9] I set $Q(z,\{\underline{z}\}) > 0$ for all $z \in Z$ in each period only for the sake of the financing constraint. Stationary equilibrium with an endogenous distribution of agents would also exist for $Q^N(z,\{\underline{z}\}) > 0$ for all $z \in Z$ in a finite number of periods $N > 0$.

the fractions of agents with the same individual state. According to this law of motion, the fraction of agents that will begin next period with assets in the set A' and a signal of ability in the set Z' is given by all those agents that transit from their current shock z to a shock in Z' and whose optimal decision for assets accumulation belongs to A'. Compared to many models in the literature on financial intermediation and occupational choice, the distribution of agents is endogenous.

Definition 1 *A stationary recursive competitive equilibrium is constant factor prices* (r, w)*, value functions* $v(a, z), v^E(a, z), v^W(a, z)$*, policy functions* $k(a, z), n(a, z), c(a, z'), a'(a, z')$*, a probability measure* λ*, transition selector* $\Delta(z, dz')$*, and aggregate levels* (A, K, L, N)*, such that*

1. *at prices* (r, w)*, the policy functions solve the optimization problem of each agent* (a, z)*,*

2. *the probability measure* λ *is time invariant,*

3. *the capital and labor markets clear,*

$$A = \int a\, \lambda(da \times dz) = \int k(a, z)\, \lambda(da \times dz) = K,$$

$$L = \int z'\, \Delta(z, dz')\, \lambda(da \times dz) = \int n(a, z)\, \lambda(da \times dz) = N,$$

4. *and the aggregate feasibility constraint holds at equality*

$$\int \{c(a, z') + \delta k(a, z)\}\, \Delta(z, dz')\, \lambda(da \times dz) =$$
$$= \int z' f(k(a, z), n(a, z))\, Q(z, dz')\, \lambda(da \times dz).$$

The aggregate feasibility constraint is implied from the other market clearing conditions by the Walras' law and that the fraction of entrepreneurs is equal to $1 - L$.

4 Characterization of Entrepreneurial Decisions

The occupational choice of each agent is based on the comparison of the expected present discounted value of each career. The following two assumptions guarantee the existence of a stationary recursive equilibrium with a positive fraction of the population in each occupation.

Assumption 1 *The signal ability shock* $\overline{z}$ *is such that there exists an asset level* a^s *for which* $\int v^W(a, z')\, \psi(dz') \leq \int v^E(a, z')\, Q(\overline{z}, dz')$ *for all* $a \geq a^s$*.*

Assumption 2 *The signal ability shock* $\underline{z}$ *is such that* $\int v^W(a, z')\, \psi(dz') \geq \int v^E(a, z')\, Q(\underline{z}, dz')$ *for all* $a \in A$*.*

Both assumptions are related to the opportunity cost of forgone wages. The first assumption requires that there be a shock sufficiently high so that agents with assets greater than a switching level a^s become entrepreneurs: the expected value of entrepreneurship is greater than the expected value of choosing to work for a wage. Vice versa, the second assumption requires a shock sufficiently low so that each agent with such a signal prefers to be a worker.

The properties of value functions for each occupation follow the analysis in Stokey, Lucas, and Prescott (1989). The value function of each occupation, $v^I(a, z')$, is strictly increasing in each argument since the utility function is strictly increasing and strictly concave and a worker's constraint set is strictly increasing in assets and the effective ability shock.

The expected value function of workers is independent of z and an increasing and continuous function of a. Due to the monotonicity of the transition matrix Q, the expected value function of entrepreneurs is an increasing and continuous function of both a and z. Finally, the value function $v(a, z)$ is non-decreasing in z and strictly increasing in a.[10]

[10] The value function $v(a, z)$—the outer envelope for the value functions at each shock level—may not be a concave function even if the value functions of workers and entrepreneurs are. Gomes, Greenwood, and Rebelo (2001) analyze a model of unemployment with a similar property. The operator on the value function satisfies the Blackwell's sufficient conditions for a contraction mapping.

Figure 2 displays values related to the occupational decision of agents with three levels of signal: low, z_L, medium, z_M, and high, z_H. As the value function of entrepreneurs is increasing in and that of workers independent of the signal ability shock, it can be easily shown that for each z there is either none or at most one switching level of assets $a^s(z)$ decreasing in z. For given prices, all agents below $a^s(z_H)$ are workers. Agents with the high signal ability shock switch to entrepreneurship early at $a^s(z_H)$, agents with the medium signal shock at $a^s(z_M)$, while agents with the lowest skill z_L never become entrepreneurs, regardless of their wealth. Thus signal z_H satisfies assumption 1 and the signal z_L satisfies assumption 2.

4.1 The Future Value of Entrepreneurship

The experience aspect contained in the monotone Markov process has important implications for the investment decisions of entering entrepreneurs. Contrary to Lucas (1978), where agents only consider the current expected incomes, it is the expected discounted present value of each career that determines an agent's occupational decision.

For a given level of signal ability shock $z \in Z$, an agent with assets at the switching level $a^s(z)$ is indifferent between working and undertaking an entrepreneurial project. Therefore, it must be the case that

$$\int v^W(a^s(z), z')\, \psi(dz') = \int v^E(a^s(z), z')\, Q(z, dz'). \tag{8}$$

The first order intertemporal condition for any asset level a and any realized effective ability shock z' is just $u_c(c(a, z')) = \beta v_a(a'(a, z'), z')$ as there is no uncertainty about the agent's next period state. Using the usual envelope conditions and assuming interior solutions, the condition (8) can be rewritten, dropping the term $(1 + r)\beta$ on both sides, as

$$\int v_a(a'(a^s(z), z'), z')\, \psi(dz') = \int v_a(a'(a^s(z), z'), z')\, Q(z, dz'). \tag{9}$$

Given the monotonicity of Q, if for some $z \in Z$

$$\int z'\, \psi(dz') < \int z'\, Q(z, dz'),$$

becoming an entrepreneur has a future value. In other words, these marginal entrepreneurs are willing to sacrifice current consumption for having the opportunity to begin their business career that brings high returns only in the future. They invest a large share of their income and wealth in order to relax the credit constraint and run their firm at the optimal size. For such agents the expected current income from business might be lower than the current expected wage,

$$\int z' w \, \psi(dz') > \int z' f(k,n) \, Q(z, dz') - wn - (r+\delta)k.$$

Because the financing constraint prevents the entrepreneur from running the firm at the optimal size, the above inequality might hold for several initial periods of entrepreneurship.[11]

4.2 The Financing Constraint

Whether entrepreneurs are financially constrained depends on their asset position, ability and the optimal size of the project measured in terms of employment level.[12] When the financing constraint is not binding at the optimal level of inputs, the hiring policy is independent of the entrepreneur's wealth and depends only on the signal ability z. Denote such unconstrained employment levels as $n^u(z)$. When the financing constraint (7) binds, entrepreneurs are not able to run a project at the optimal size and their hiring decisions depend only on their wealth. The binding financing constraint can be solved for a level of $n^b(a)$ independent of the signal ability shock. In general, an optimal hiring policy must satisfy

$$n(a,z) \leq \min \left\{ n^u(z), n^b(a) \right\}.$$

The actual employment size is a function of the expected marginal utilities and productivity for all levels of the effective ability shocks.

In Figure 3, the unconstrained policies are horizontal lines $n^u(z)$, while the credit-constrained employment level lies on the increasing concave function $n^b(a)$. Agents

[11] In the search model with occupational choice by Gomes, Greenwood, and Rebelo (2001), consumption of searchers similarly decreases compared to workers who keep their jobs.

[12] Similar results apply to the size of firms measured in terms of capital input. As in Lucas (1978) it can be easily shown that all all entrepreneurs use the same optimal capital-labor ratio.

with a low signal shock are always workers with $n(a, z_L) = 0$. Entrepreneurs with the medium signal shock are unconstrained as they run firms of a small size. Finally, the employment policies of entrepreneurs with the high ability signal and assets beyond the switching level $a^s(z_H)$ must lie below the thick line $n(a, z_H)$. The poorer entrepreneurs with the high signal shock cannot hire the optimal employment level $n^u(z_H)$ but must use at most a lower, inefficient level $n^b(a)$. If they received a bequest or inherited additional assets, their projects would increase towards the optimal size (as in Holtz-Eakin, Joulfaian, and Rosen (1994) empirical study).

5 Quantitative Analysis

The parameterization shown in Table 1 is standard for the U. S. economy as in Cooley (1995). The span of managerial control θ set at 0.92, a level close to under 1 estimated by Burnside (1996). The utility has the logarithmic form.

The transition matrix for entrepreneurial skills has important implications for the degree of business persistence and accumulation of wealth by business families. I set the values of Q and the levels of shocks Z so that the model is able to replicate the first and second moments of the distribution of wealth. Similarly to Veracierto (2001), I choose the effective ability shocks for the entrepreneurs $Z = \{0\} \cup [1/2, \overline{z}]$ with $Q(\{0\}, \{0\}) = 1$ so that an entrepreneur who fails with the lowest effective ability shock will prefer to be a worker in the following period. Also, $Q(z, \{0\}) > 0$ for all $z \in Z$ implies that all entrepreneurs terminate their businesses in finite time. The workers draw their effective ability shocks from a fixed distribution ψ with a lowest possible value equal to one half. It is as if the workers became unemployed with a low probability and received unemployment benefits with such a replacement ratio. This specification of shocks and their laws of motion imposes the financing constraint in each period and satisfies the assumptions on the existence of a stationary equilibrium. Both transition processes are specified so that the entrepreneurs constitute 9% of the population and the exit and entry rate is around 4.5%, a number estimated by Evans (1987).

The algorithm for finding the steady state of each regime is relatively simple. To solve for the occupational decision, expected values of both options are computed first. For a given interest rate, I iterate on the wage and the interest rate until both markets are cleared with the optimal policies and the endogenous stationary distribution obtained at these prices.

5.1 Stationary Equilibrium

Table 2 shows the aggregate levels, prices, and distribution of agents and resources in the steady state. The proportion of entrepreneurs is 9%, matching the data for the U.S. economy. Importantly, this model with of occupational choice and entrepreneurial investment can replicate the more concentrated distribution of wealth (measured by assets) with Gini coefficient 0.83. Wealth inequality is present also within each occupation (0.62 for entrepreneurs and 0.80 for workers). The Gini coefficient of income inequality is 0.48. The top percent of the agents own 27% of the total wealth and receive 8% of the total income. The top 5% own more than half of the total wealth and receive one quarter of the total income. As in the data analyzed by Quadrini (1999a), the entrepreneurs are heavily over-represented in the top wealth docile, with 50%, as well as in the second top docile, with 25%.

The shares of wealth owned and of the total income received by entrepreneurs, 37% and 26%, are almost the same as in the data, 40% and 22%, respectively. Thus in this model the entrepreneurs hold on average six times more assets, receive four times more income, and consume two times more than the workers. The total credit in the economy (defined as net savings of entrepreneurs and workers) is 203% of the output, close to the same statistics for the most developed countries. Entrepreneurs use on average 2.65 times more capital than they own assets. The capital-output ratio is 3.2 and agents consume 80 percent of the total output. The wealth/income ratio is 4.57 for entrepreneurs and 2.68 for the workers (the first one is lower than in the data reported by Quadrini (1999a)).

The average return on entrepreneurial project is 6.37%, twice as much as the risk-free interest rate, 3.2%.[13] The unconstrained entrepreneurs produce 41% of total

[13]For the evidence on the effects of firm size on interest rate see Petersen and Rajan (1994). As in Huggett (1997), the heterogeneity of agents together with the borrowing constraint lead to over

output and employ the same fraction of workers.[14]

The future value of entrepreneurial career is significant. Taking into account the net capital gains accruing to the business, Hamilton (2000) finds that the median entrepreneurial earnings are initially 35% lower than those of the workers. He also finds that entrepreneurial earnings tenure profile overtakes that for the wage after a few years in business. The median entrant in my model follows the same path. This is illustrated by Figure 4 which displays the asset accumulation, total income, investment, consumption and the return on the business project of the median entrant.[15]

In Figure 4, in the first two periods the agent is worker and enters entrepreneurship in period 3. The total income of the entrant falls by 35% and overcomes the previous workers' income only after three periods. The agents has high entrepreneurial skills: the business is very profitable with returns reaching almost 20%. This requires a massive, lumpy investment: the agent invests from 50 to 60 percent of the total income and above 30% of his assets (prior to entering, the agent was investing around 3% of his total income). This investment is used to relax the financing constraint so that the business can operate at the optimal size corresponding to this entrepreneurial skill. For several periods, the firm grows at a rate of 15-20%. The cost of this business expansion is the dramatic drop in consumption in the first six periods of entrepreneurship.[16]

Thus the median entrant chooses self-employment despite a significantly lower current expected income than that available as a paid employee. Without any non-pecuniary benefits of self-employment, the positive compensating differential for the risk-averse business owners arrives only after the sacrifice during the first initial periods. After six periods in business, the median entrant already owns ten times more

savings in the sense that the equilibrium interest rate lies below the rate of time preference.

[14]As I have omitted the analysis of large corporations, these entrepreneurs can be interpreted as firms with unconstrained access to credit markets.

[15]I look at the median entrant to avoid the "superstar" bias. For the first two periods I follow the agent with median assets with the average workers' shock, $z = 1$. Then in the third period the agent receives the second highest ability shock and enters entrepreneurship (at this asset level 95% entrants enter with this shock). From then on I plot the average (over 20,000 simulations) path for entrepreneurs starting from this individual state.

[16]This is despite the absence of tenure increase for the wage process.

assets and receives three times higher total income than the median agent.

Over time the successful entrepreneurs accumulate significant amount of savings, become unconstrained and operate efficient firms. Again, this behavior is consistent with the data. Quadrini (1999a) uses the PSID data to observe that while worker families tend to stay in or move to lower positions of wealth, business households tend to stay in or move to higher positions. Finally, he finds that the longer the business life, the higher the wealth accumulated by business families. Undertaking of entrepreneurial activity is an important way for households to switch to higher classes of wealth.

As wealth is important determinant of occupational and business decisions there should be a strong correlation between wealth and entrepreneurship. Figure 5 displays the probability that an agent with an accumulated asset level is an entrepreneur. Paulson and Townsend (2002) document a very similar shape of the likelihood function for the Thai economy. As exiting entrepreneurs keep to a large extent their accumulated wealth, financial constraint makes the reentry to entrepreneurship another reason why business families accumulate higher levels of wealth relative to workers.[17]

6 Conclusions

This model shows that modelling occupational choice of heterogenous agents and the investment decisions of wealth-constrained entrepreneurs is important for matching the U.S. distributional data. In particular, saving and investment decisions under limited enforceability of contracts are capable of generating the very unequal distribution of wealth. The main factor behind the high levels of savings of business families is the incentive to overcome the financing constraint that prevents them from operating their firms at the optimal size.

These results are important for the crucial role small firms play in innovation,

[17]Quadrini (1999a) shows that the entrance rate for business owners with previous experience is 23%, and without 2.6%. Notably, 64% of owners have previous work experience in a managerial capacity and 34% as the owner of another business.

technological change and productivity growth. In the 1990s, small businesses employed more than half of the workforce and created three-fourths of the new jobs. As these firms are often financially constrained, government policies targeting the entry of financially constrained and skilled entrepreneurs can have significant impact on occupational choice, efficiency, aggregate levels and the distribution of resources. Bohacek and Rodriguez-Mendizabal (2003) and Kiyotaki and Moore (2001) look at the real effects of monetary policy on liquidity constrained agents with occupational choice. The framework developed in this paper can also be used to study the effect of technology innovations on entry and exit into entrepreneurship. In a related paper, Jermann and Quadrini (2003) analyze the impact of technology innovations in a related model with heterogeneous firms with limited enforceability of financial contracts.

There are many important issues the paper does not address. Although moral hazard related to entrepreneurial skill, effort, accumulated assets, or realized profits is not necessary to deliver the empirical facts studied in this paper, it might have significant incentive effects on entrepreneurial decisions (see Cooley, Marimon, and Quadrini (2001) for an excellent example). Also, financial intermediation can be modelled in a greater detail. The banks could optimally provide multiperiod loans, break even only in expectation, or require collateral that fits the needs and characteristics of the entrepreneurs. Consequently, the financial contracts could be optimally designed to provide not only capital for the entrepreneurial project but also some insurance against its failure. These issues are left for future research.

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Parameters of the Model

Technology and Preferences

α	θ	δ	β
0.34	0.92	0.06	0.95

Ability Shocks Z

0	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00

Distribution ψ of Workers' Ability Shocks

0	0.05	0.80	0.05	0.04	0.03	0.02	0.01	0

Transition Matrix Q for Entrepreneurs' Ability Shocks

1	0	0	0	0	0	0	0	0
0.2250	0.7500	0.0250	0	0	0	0	0	0
0.1000	0.0750	0.8000	0.0250	0	0	0	0	0
0.0500	0	0.0750	0.8500	0.0250	0	0	0	0
0.0250	0	0	0.1000	0.8500	0.0250	0	0	0
0.0125	0	0	0	0.1125	0.8500	0.0250	0	0
0.0125	0	0	0	0	0.1125	0.8500	0.0250	0
0.0125	0	0	0	0	0	0.1125	0.8500	0.0250
0.0125	0	0	0	0	0	0	0.1375	0.8500

Table 1: Parameters of the model.

Aggregate Levels, Prices, Ratios			
	Average		
	All	Entrepreneur	Worker
Output	8.14	90.80	—
Income	8.14	24.72	6.51
Consumption	6.55	12.88	5.93
Investment	1.59	11.84	0.58
Assets	26.06	113.12	17.45
Wealth/Income	3.20	4.57	2.68
Capital/Wealth	—	2.65	—
Interest rate	0.032	—	—
Wage	4.26	—	—

Distribution of Wealth and Income						
	Top Percentiles (in %)					Gini
	1%	5%	10%	20%	30%	Index
Wealth						
SCF 1989	36	58	70	84	92	0.86
PSID 1989	29	50	63	78	88	0.77
Model Economy	27	59	73	86	92	0.83
Income						
SCF 1989	17	32	42	57	69	0.54
PSID 1989	8	20	32	48	61	0.45
Model Economy	8	25	39	55	66	0.48

Occupational Shares (in %)				
	Population	Wealth	Income	Consumption
Entrepreneurs	9	37	25	17
Workers	91	63	75	83

Table 2: Steady state aggregate levels and distribution of resources

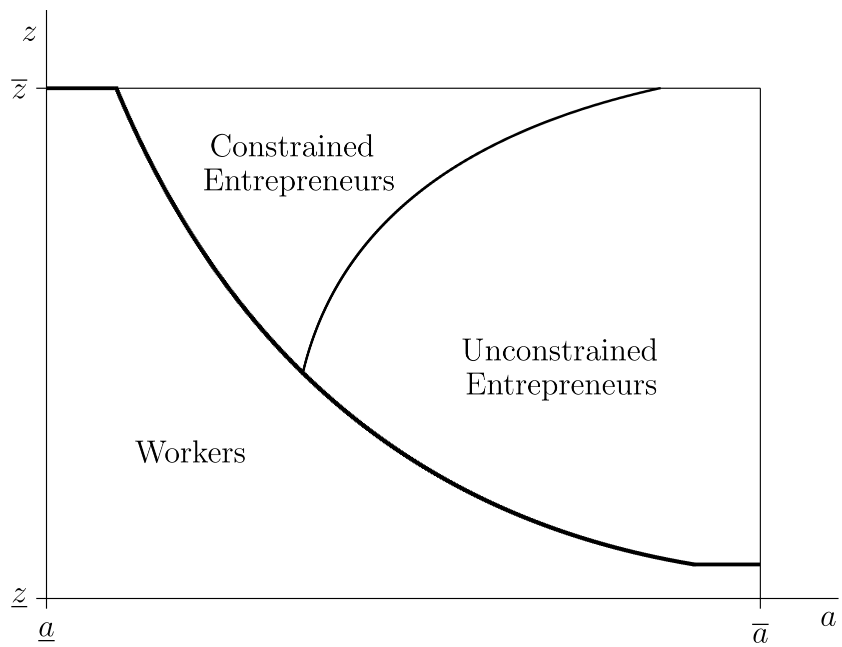

Figure 1: Occupational choice as a function of assets and entrepreneurial skill.

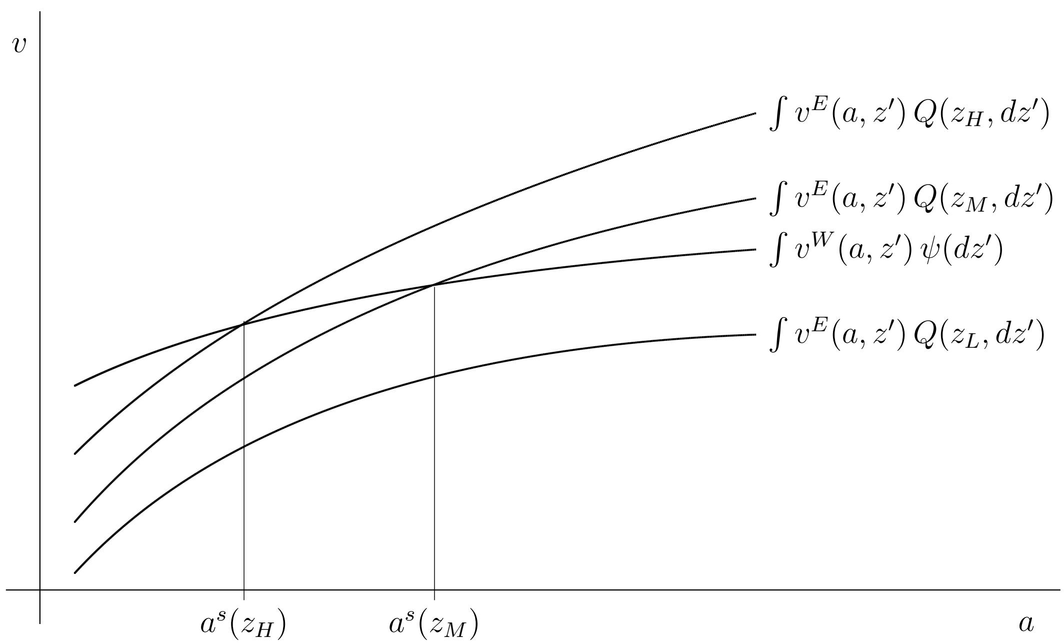

Figure 2: Value functions of entrepreneurs and workers

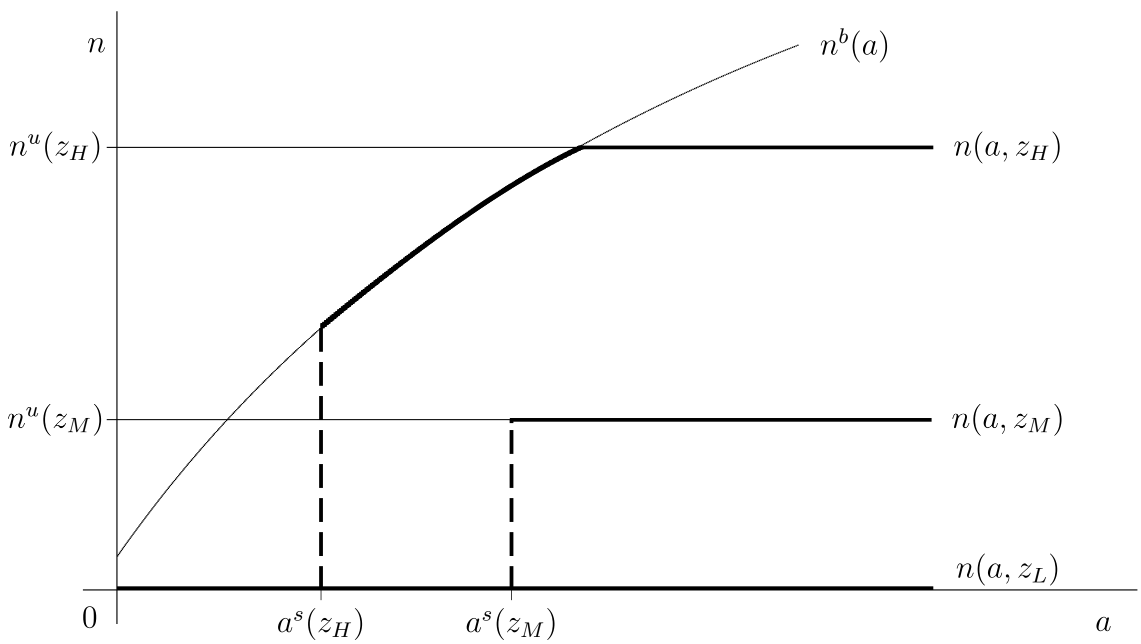

Figure 3: Employment policies of entrepreneurs.

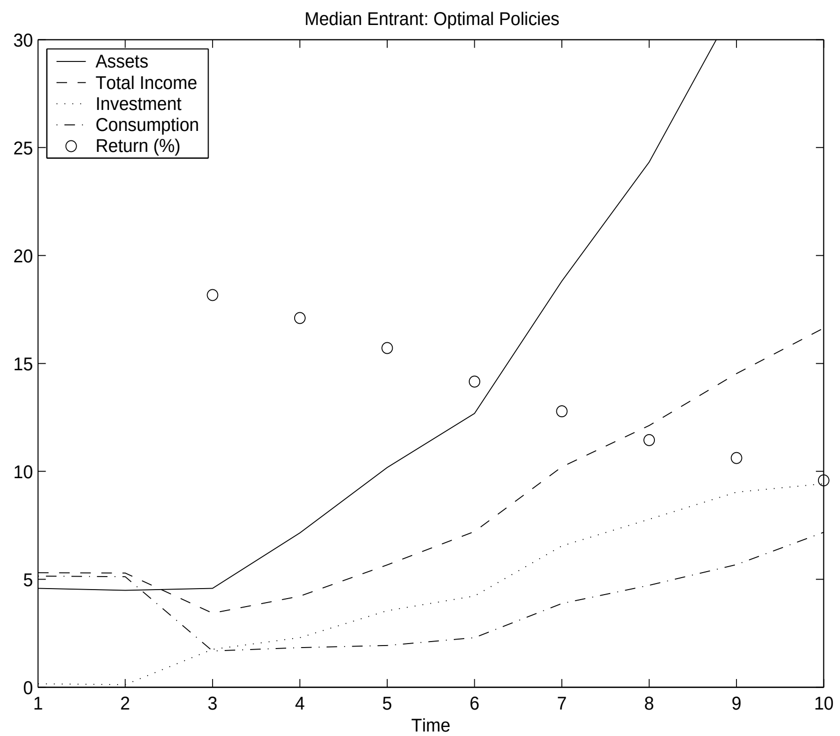

Figure 4: Optimal policies of the median-wealth agent. Occupation: worker in periods 1-2, entrepreneur from period 3.

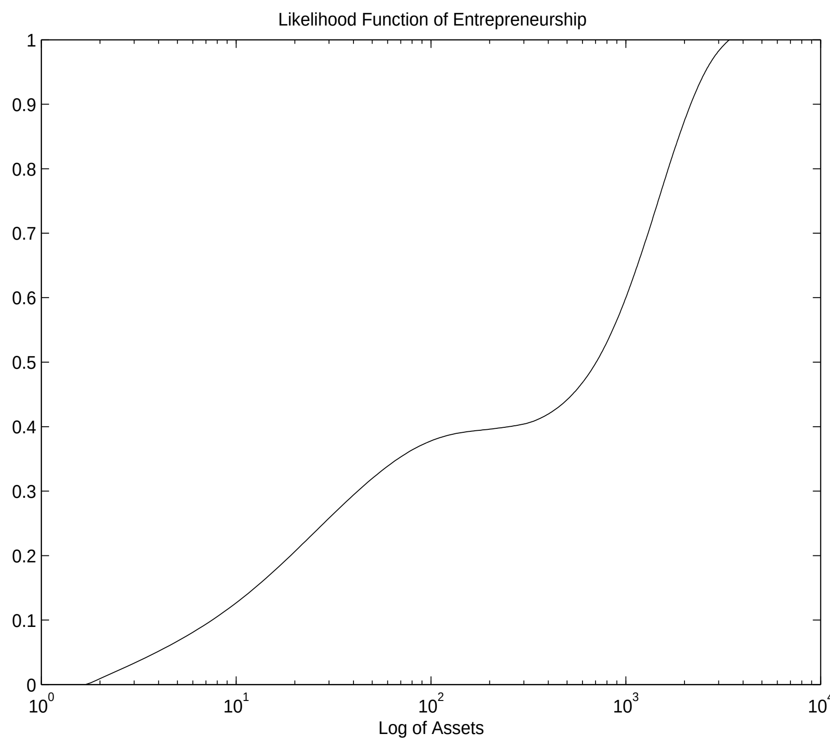

Figure 5: The likelihood of agent's entrepreneurship as a function of assets.