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Macro III
Spring 2007

Homework 6

Due July 11, 2007

1. *Intermediate Inputs as Durables.* Suppose that the intermediate inputs, X_{ij}, are infinite-lived durable goods rather than non-durables as we had in class. New units of these durables can be formed from one unit of final output. The inventor of the jth type of intermediate good charges the rental price R_j, and the competitive producers of final goods treat R_j as given.

   (a) How is R_j determined?

   (b) In the steady state, what is the quantity, X_j, of each type of intermediate good?

   (c) What is the steady-state growth rate of the economy? How does this answer differ from the case we had in class in which the intermediate inputs were perishable goods?

   (d) If the intermediate goods are durables, then what kinds of dynamic effects arise in the transition to the steady state?

2. *An Endogenous Innovation Model a la Jones (1998).* Consider the Jones model from Jones (1998) Introduction to Economic Growth, Ch.5. Accordingly to that there are theree sectors in the model economy:

- production function in the competitive final goods sector: $Y_i = L_{Y,i}^{1-\alpha} \sum_{j=1}^{A} X_{ij}$ with $L_{Y,i}$ being labor input and X_{ij} being the durable intermediate good j input;
- production function in the intermediate good j sector: the service from one unit of final (raw) capital good with rental price r is transformed into one unit of the intermediate capital good j which is rented to the final good sector at rental price R_j;

- production function in the R&D sector for the change in the stock of new ideas A: $\dot{A} = \bar{\delta} L_R$ where L_R is the labor used in the R&D sector and $\bar{\delta} = \delta L_R^{\lambda-1} A^{\phi}$ with given constants $\delta, \lambda < 1$ and $0 \leq \phi < 1$.

1. (a) *An increase in the productivity of research.* Let's assume that $\lambda = 1$ and $\phi = 0$, then suppose there is an unexpected permanent one-time increase in the productivity of research, represented by an increase in δ. What happens to the growth rate and the level of technology over time?

   (b) *Too much of a good thing?* Show that the level of knowledge along a balanced growth path is given by $A^*(t) = \frac{\delta L_R(t)}{\gamma_A^*}$, where γ_A^* is the balanced path growth rate of technology - explain. Using this fact derive the expression for the level of output per capita along a balanced growth path $Y^*(t)$. Find the value of $s_R \equiv \frac{L_R}{L}$ that maximizes output per capita along a BGP for this case. Why is it possible to do too much R&D in the decentralized economy according to this criterion?

   (c) *The future of economic growth.* According to Jones (1997) ["The Upcoming Slowdown in U.S. Economic Growth." NBER Working Paper No. 6284, National Bureau for Economic Research] (see also Ch.4 in Jones (1998)) the number of scientists and engineers engaged in R&D has been growing faster than the population growth in the advanced countries of the world. To take some plausible numbers, assume population growth is 1 percent and the growth rate of reseachers is 3 percent per year. Assume that $\dot{A}/A$ has been constant at about 2 percent per year. (Why?)

      i. Using the fact that $\lambda \left(\frac{\dot{L_R}(t)}{L_R(t)} \right) - (1-\phi) \left(\frac{\dot{A}(t)}{A(t)} \right) = 0$ as far as $\left(\frac{\dot{A}}{A} \right)$ and $\left(\frac{L_R^{\lambda}}{A^{1-\phi}} \right)$ are constant (prove it), calculate an estimate of $\lambda/(1-\phi)$.

      ii. Using this estimate and equation for γ_A^*, calculate an estimate of the long-run steady-state growth rate of the world economy.

      iii. Why are these numbers different? What do they mean?

      iv. Does the fact that many developing countries are starting to engage in R&D change this calculation?