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Macro III  
Spring 2007

## Homework 5

Due June 27, 2007

*Human Capital Externalities in an Endogenous Growth Model.* [This is based on Lucas paper published in Journal of Monetary Economics in 1988] Consider the model of Section 4 in Lucas (1988) given by the goods production constraint (11) and the education technology (13).

1. (a) Reformulate the model for the decentralized economy as we had in class where households rent physical and human capital to perfectly competitive firms producing goods. Human capital is self-produced according to (13). Derive the first-order conditions and the balanced growth path values of growth rates for output, physical and human capitals and consumption. Express the model equations in transformed variables  $q = \frac{c}{k}$ ,  $x = \frac{k}{h \frac{1-\beta+\gamma}{1-\beta}}$  and  $u$  which exhibit the property of steady state with zero growth in variables  $q, x$  and  $u$  (low letter variables  $c, k, h$  are the values in per capita terms).
- (b) Derive the socially optimal growth rate and compare it with the decentralized one. Explain the difference and its source.
- (c) Reformulate the model in continuous time from (a) into a discrete time form, i.e. e.g. the change in physical capital  $\dot{K}_t$  will be now  $K_{t+1} - K_t$ . Answer all questions raised in (a).