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Macro III

Summer 2007

Homework 4

Due June 20, 2007

1. *Adjustment Costs in the Ramsey Model.* Consider the model of adjustment costs we had in class. Assume that consumers have the usual Ramsey preferences. But, instead of assuming a constant interest rate, consider the equilibrium for a closed economy.

   (a) Find an expression for for $\dot{q}$ as a function of q, i/k, $\dot{c}/c$, and k.

   (b) Use a phase diagram to work out the dynamics of i/k and k. [It is easier to work with i/k than with q.]

2. *Capital Externalities in a Neoclassical Growth Model.* Consider the usual Ramsey-Cass-Koopmans model as discussed in class (i.e. with population growth at rate n and exogenous technological progress at rate g). Assume instantaneous utility is CRRA. Let final output be produced by a continuum of (identical) firms of mass one (firms can be continuosly indexed on the interval $i \in [0, 1]$), with each firm using the same production function

$$Y_i = \kappa^\beta K_i^\alpha (AL_i)^{1-\alpha},$$

where $\kappa = \int_0^1 K_i di$. Assume that there are positive externalities to capital accumulation. That is, firms take the aggregate capital stock, κ, as given at each point in time. Notice that because the measure of firms is one, $\kappa = K = K_i$. The rest of the model is identical to the Ramsey framework.

   (a) What is the per capita growth rate of the economy along the balanced growth path? If $g = 0$ is there per capita growth? Why?

   (b) Solve for the investment rate $s = I/Y$ along the balanced growth path for the decentralized economy.

   (c) What is the socially optimal investment rate along the balanced growth path, and why are these rates different?