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Macro III

Summer 2007

Homework 3
Due June 6, 2007

1. *A discrete-time version of the Ramsey Model.* Consider the basic setup of the Ramsey model discussed in class, however, with discrete time. The representative household solves
$$V(v_0) = \max_{\{c_t\}} \sum_{t=0}^{\infty} \left(\frac{1+n}{1+\rho}\right)^t u(c_t)$$
subject to
$$v_{t+1} - v_t = w_t + r_t v_t - n v_t - c_t$$
where $u(c)$ has a constant intertemporal elasticity of substitution given by θ^{-1}, $\rho > n > 0$. Assume output per capita is produced according to the production function $y_t = k_t^\alpha A_t^{1-\alpha}$ where A is exogenously growing at rate $g > 0$, k_t is the level of capital p.c. at the beginning of period t and assume that capital depreciates at rate $\delta > 0$.

   (a) Formulate the Bellman equation for the household problem and derive the first order conditions. Do we need the NPG condition in this formulation?

   (b) Using the flow budget constraint derive the intertemporal budget constraint.

   (c) Solve the problem of the representative perfectly competitive firm producing output.

   (d) Solve the model to find a discrete-time version of the Euler equation for consumption per effective labor and the steady-state ratio of capital per capita to the level of technology.

   (e) Show the dynamics of the system in a phase diagram.

2. *Distortionary taxation in the discrete-time Ramsey Model.* Consider the same model as in 1. above but additionally we introduce the government which taxes capital and returns the revenue it collects from the taxation via lump-sum transfers to households.

(a) Derive the FOCs for households and firms and get the Euler equation.

(b) Suppose, starting from steady state, there is a permanent, unanticipated decrease in the tax rate to $\tau' < \tau$. Analyze the change in a phase diagram. Be sure to show how consumption evolves over time.

(c) Suppose instead that the change in the tax rate is *anticipated*, i.e. it is announced one year in advance, and it is only *temporary,* i.e. it will last only four years. Analyze this change in a phase diagram. How and why is the consumption path different?

(d) Suppose there are many economies like this one. Household's preferences are the same in each country, but the tax rates on capital income may vary across countries. Assume that each country is on its balanced growth path.

   i. Show that the saving rate on the balanced growth path, $(y^* - c^*)/y^*$, is decreasing in τ.

   ii. Do citizens in low-τ, high-k^*, high-saving countries have any incentive to invest in low-saving countries? Why or why not?

   iii. How your answer to (ii) will change if countries can also differ in their citizens' preference parameters?