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Macro III
Summer 2007

Homework 1

Due May 23, 2007

1. Consider the standard Solow-Swan model:

$$\begin{aligned} Y(t) &= F(K(t), A(t)L(t)) \\ \dot{K}(t) &= sY(t) - \delta K(t) \\ \dot{L}(t)/L(t) &= n \\ \dot{A}(t)/A(t) &= g_0 \end{aligned}$$

where $F(\cdot)$ exhibits constant returns to scale in K and L, and $1 > s > 0$, $n > 0$, and $g_0 > 0$.

   (a) Characterize the steady state for the model and show that it is stable. Let $x \equiv X/L$ and $\tilde{x} \equiv x/A$ for any variable X.

   (b) Suppose the economy is in steady state and the rate of technological progress jumps permanently and immediately to $g_1 > g_0$. Characterize the new steady state and the transition dynamics in a Solow diagram. What is the intuition for the behavior of y/A and k/A?

   (c) Show graphically what happens to the growth rates of the capital-labor ratio and per capita output over time. Be sure to pay close attention to the transition dynamics.

   (d) Show graphically what happens to the natural log of the capital-labor ratio and per capita output over time.

2. *Intertemporal and Dynamic Budget Constraints.* Consider the following intertemporal budget constraint:

$$\int_t^\infty C_s e^{-\bar{r}_s(s-t)} ds = V_t + \int_t^\infty (Y_s - T_s) e^{-\bar{r}_s(s-t)} ds$$

where C is consumption, Y is labor income, T is a lump-sum tax, V is financial wealth, and $\bar{r}_s = 1/(s-t) \int_t^s r_u du$. Derive the dynamic budget constraint for this consumer. [Hint: You have to differentiate with respect to t both sides of the equation. Check the Appendix 1.5.6 in Barro&Sala-i-Martin's book how to differentiate integrals]