Macroeconomics III
CERGE
Michal Kejak
January 2002

MACRO GENERAL EXAMINATION PART III A

(28)

1. (45 points) Consider the Solow growth model with the exogenous growth rate of technological progress $\dot{B}/B = \gamma$ and the rate of population growth $\dot{N}/N = n$ described by

$$\dot{K}/K = sF(K, L)/K - \delta \tag{1}$$

where $F(K, L) = BK^{1-\alpha}L^{\alpha}$, $0 < \alpha < 1$, s is fixed saving rate, and $\delta > 0$ is the rate of depreciation.

   (a) (16) Derive the equation for the growth rate of capital per effective labor $g_{\hat{k}}$, where $\hat{k} = K/(BL)$.

   (b) (8) Derive the steady-state condition for the level of capital per effective labor and the level of output per effective labor. What is the growth rate of output at steady state?

   (c) (16) Using a log-linear approximation of the growth equation for $\hat{k}$ from (1a) in the neighborhood of the steady state we obtain

$$g_{\hat{k}} = d\ln(\hat{k})/dt \cong -\beta[ln(\hat{k}/\hat{k}^*)].$$

   What is the expression for β?

   (d) (5) What is σ convergence?

Macroeconomics
CERGE
Michal Kejak
Summer 2003

MACRO GENERAL EXAMINATION PART II

Read everything carefully before you start! This part of exam is worth of 70 points alltogether. GOOD LUCK!

Romer's Model with Knowledge Spillovers. Consider a version of the Romer growth model with knowledge spillovers with a continuum of (identical) perfectly competitive firms of mass one (firms can be continuously indexed on the interval $i \in [0, 1]$). The firm i has the production function $Y_i = K_i^{\epsilon} L_i^{1-\epsilon} K^{\eta}$ where $1 > \eta, \epsilon > 0$, K_i is capital input, L_i is labor input, Y_i is the firm i output and $K = \int_0^1 K_i di$ is the aggregate capital. There is a continuum of identical households who rent their capital to the firms and inelastically supply one unit of their labor to them. Assume for simplicity there is no population growth, the size of population is equal to one, there is one worker per firm, agents are not allowed to borrow/lend among themselves, and the depreciation rate is equal to zero.

1. *Standard Model.* An infinitely-lived household maximizes its life-time utility

$$\int_0^{\infty} \frac{1}{\alpha} c_t^{\alpha} e^{-\rho t} dt$$

with $-\infty < \alpha < 1$.

(a) (10) Derive the expression for the intertemporal elasticity of substitution in consumption.

(b) (10) Set up a representative household's optimization problem. What are the state and control variables? What is the current-value Hamiltonian? Derive the first order conditions.

(c) (10) Assuming perfect competition framework in production factor markets derive the expression for the interest rate and the wage rate.

(d) (10) Derive the Euler equation. What is the growth rate of consumption? Discuss briefly the situation when $\epsilon + \eta > 1$ and $\epsilon + \eta < 1$. For the rest of this question assume that $\epsilon + \eta = 1$. Compute the growth rates of consumption, of capital, and of output and the level of the consumption-capital ratio $\mu \equiv C/K$ as a functions of the model parameters only. Compute the saving rate of the economy.

2. *Utility Enhancing Government Expenditure.* Assume further that the government expenditure G, which is a fixed fraction $0 < g < 1$ of aggregate output, $G = gY$, contributes to the welfare of the private agent. Thus the instantaneous agent utility function is now $u(c_t, G_t) = \frac{1}{\alpha}\left(c_t G_t^{\beta}\right)^{\alpha}$ with $-\infty < \alpha < 1$ and $\alpha\beta < 1$. The government expenditure is financed by taxes on household income and on consumption given by constant tax rates τ_y and τ_c. To keep the government budget balanced at any moment there is additionally lump-sum transfer to household T. Thus, the government budget constraint is $G + T = \tau_y rK + \tau_y w + \tau_c C$.

   (a) (10) *Decentralized Equilibrium.* Set up this modified problem of a representative household. What are the state and control variables? What is the current-value Hamiltonian? Derive the first-order conditions. Using the fact that the consumption-capital ratio is constant, i.e. $C/K = \mu$, derive the Euler equation. Compute the growth rates of consumption, of capital, and of output and the level of the consumption-capital ratio μ as a functions of the model parameters only. Compute the saving rate of the economy. What is the expression for μ when the preferences are logarithmic? Discuss all the results. [Hint: Use the economy resource constraint to derive μ.]

   (b) (10) *Social Optimum.* Set up the social planner problem. What are the control variables? Derive the first order conditions. Compute the optimal growth rates of consumption, of capital, and of output and the level of the consumption-capital ratio μ as a functions of the model parameters only. Compute the optimal saving rate of the economy.

   (c) (10) Compare the decentralized equilibrium results above with those of the optimum. What are the sources of inefficiency (Pareto non-optimality) of the decentralized equilibrium in this model. Can you use the fiscal policies introduced in the decentralized equilibrium model above to obtain the socially optimal allocation of resources? Discuss why or why not.

SUMMER 2005 MACRO GENERAL EXAM PART IB

Read everything carefully before you start! This part is worth of 35 points altogether. GOOD LUCK!

1. Consider a version of Romer's economy with knowledge spillovers where the representative household maximizes its lifetime welfare subject to its flow budget constraint and the No-Ponzi-Game condition. Assume that the utility function is of CRRA type and that the size of the household grows at rate $n > 0$. There is a continuum of (identical) perfectly competitive firms of mass one (firms can be continuously indexed on the interval $i \in [0, 1]$). The firm i has the production function $Y_i = AK_i^{\alpha}L_i^{1-\alpha}K^{\alpha}$ where $1 > \alpha > 0$, K_i is capital input, L_i is labor input, Y_i is the firm i output and $K = \int_0^1 K_i di$ is the aggregate capital. Assume further that there is a government which taxes household consumption and labor income at constant rates, τ_c and τ_w, respectively; and that the firms pay taxes from renting capital at constant rate τ_r. Let the government consumes the amount of the tax revenues from taxing consumption and the rest is returned back to the households in the form of lump-sum transfers.

   (a) Write down the consumer and government budget constraints.

   (b) Solve for the growth rates, c, y, and k along the balanced growth path.

   (c) Which taxes can affect the long-run growth rate and which cannot? Explain.

   (d) Discuss a combination of taxes and subsidies that the government could employ that will maintain a balanced budget and simultaneously implement the socially optimal allocation of resources.