Final Examination

Read everything carefully before you start! The exam is worth of 180 points. GOOD LUCK!

1. (90 points) *The Ramsey Growth Model* Consider an economy a la Ramsey with infinitely lived representative households, who provide labor services in exchange for wages w_t, receive interest income on their capital $r_t k_{t-1}$, purchase goods for consumption c_t, and decide on capital k_t and money holding M_t. Money are necessary for purchasing goods, $M_{t-1} + V_t \geq P_t c_t$, where V_t is money transfer from the Central Bank and P_t is the price level. Let the representative household lifetime welfare is be

$$\sum_{t=0}^{\infty} \frac{c_t^{1-\theta}}{1-\theta} \left(\frac{1}{1+\rho} \right)^t$$

where $\rho > 0$ is the subjective discount rate. Assume further that there is constant number of households normalized to $L = 1$ and that the government purchases per capita g_t, which are a complete waste, are financed via income tax (taxed income is income on working and income on interest) with the flat tax rate $0 < \tau < 1$ in such a way that the government budget is balanced at any moment of time. Let there be the Central Bank which prints money at the constant rate $\sigma = \frac{M_t - M_{t-1}}{M_{t-1}}$ and transfers them to the households. The production sector of the economy is again according to the Ramsey model composed of representative perfectly competitive firms which produce goods, pay wages for labor input, and make rental payments for capital inputs. The firms have the neoclassical production function, expressed in per capita terms, $y_t = A k_t^{\alpha}$, where $0 < \alpha < 1$, and capital depreciates at the rate $\delta > 0$.

(a) (10) Write down the household's budget constraint.

(b) (10) Write down the consolidated government budget (this is when the Central Bank is considered as a part of the government).

(c) (10) Specify the household's dynamic optimization problem. What are the state and control variables?

(d) (15) Using the Bellman equation derive the first order and envelope conditions of the household's optimization problem.

(e) (10) Derive and explain the expression for μ_t/λ_t where λ is the shadow price of real income and μ is the shadow price of holding money in advance.

(f) (10) Derive and explain the Euler equation.

(g) (10) Write down and solve the problem of a profit-maximizing representative firm. Using the results above specify the competitive market equilibrium.

(h) (15) Derive the conditions for the steady state level of capital and consumption per capita. Do they depend on government policies? Explain why or why not. Is money at steady state superneutral and/or neutral? Explain. What is the steady state inflation rate?

2. (20) What is the relation among absolute, conditional, and club convergence?

3. (70 points) Consider the Romer (1990) version of the model with a variety of producer products, as we had in class, with the production function for firm i given by

$$Y_i = L_{Y,i}^{\beta} \sum_{j=1}^{A} X_{ij}^{1-\beta}$$

where $0 < \beta < 1$, Y_i is output, $L_{Y,i}$ is labor input, and X_{ij} is the employment of the jth type of specialized intermediate (nondurable) output.

(a) (10) Derive the final good's sector demand for labor and intermediate goods as a function of the prices w and P_j.

(b) (10) We assume that the inventor of good j retains a perpetual monopoly right over the production and sale of the good, $\underline{X_j}$. As in class, one unit of final good combined with a design can be transformed into one unit of intermediate good. Solve the profit maximization problem for an intermediate-goods firm that owns a patent and show that all intermediate goods sell for the same price. What is the instantaneous profit earned by an I-firm as a function of labor used in the final-good sector and of the <u>stock of designs</u>?

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(c) Assume that the production function in R&D sector for discovering new ideas is more general than we had in class and is given by $\dot{A} = \hat{\phi} L_A$ where $\hat{\phi} = \frac{1}{\eta} L_A^{\lambda-1} A^{\epsilon}$ captures the externality effects of the stock of ideas A and the number of people searching for new ideas L_A. Assume that $\epsilon < 1$ and $0 < \lambda < 1$.

  i. (15) Using the above general production function specify the rate of technological change $\frac{\dot{A}}{A}$ as a function of labor used in R&D sector and of the level of technology? Derive the expression for the rate of technological change along a balanced growth path. Is this model a true endogenous growth model? Why or why not? [Hint: Assume that along a BGP $(\frac{\dot{L}_A}{L_A})^* = n.$]

  ii. (10) What is the rate of technological change along a BGP if $\epsilon = 1$ and $\lambda = 0$?

  iii. (10) What we mean by a scale effect in economic growth theory? Looking at your result in (ii) how you can introduce the scale effect into this model?

(d) (15) Explain in words the nature of all market failures (inefficiencies) that emerge from the decentralized model.

Macroeconomics III
CERGE
Michal Kejak
Summer 1998

Final Examination

Read everything carefully before you start! You are not required to solve BONUS questions! The exam is worth of 180 points alltogether without bonuses. GOOD LUCK!

1. (60 points) Consider an infinite horizon model with representative agent (Ramsey-Cass-Koopman model) and the following characteristics: logarithmic utility, Cobb-Douglass production function, population growth $n > 0$ and exogenous growth of technology $x > 0$. Suppose that the government taxes the return to capital at the rate τ and returns the revenue it collects from this tax through lump-sum transfers.

   (a) (10) Set up a representative agent's optimization problem.

   (b) (10) Derive the first order conditions.

   (c) (10) Derive and explain the Euler equation.

   (d) (10) Derive steady state capital per unit of effective labor in terms of the parameters of the model. Explain the effect of the tax rate τ on the steady state capital per unit of effective labor.

   (e) (10) Suppose now that the economy is in a steady state and that there is an unexpected, permanent decrease in the tax rate $\tau' < \tau$. Show the effects of the tax decrease in the phase plane $(\hat{k}, \hat{c})$ where ($\hat{\ }$) denotes variables expressed per unit of effective labor.

   (f) (10) What policy can the government choose to reach the social optimum?

2. (40 points) Choose one of AK models we had in class, except the simplest one with the firm production function $Y = AK$. Write and describe the production function of a typical firm and explain what are the assumptions that eliminate the tendency for diminishing returns. Explain in words the nature of market failures (inefficiencies) that emerge from decentralized competition in the model. What sorts of policies could be enacted by a government in order to correct these market failures? Are there any scale effects in the model? Explain why or why not.

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3. (30 points) Assume that the growth rate of capital per effective labor, $\hat{k}$, is given as

$$\gamma_{\hat{k}} = sA\hat{k}^{-(1-\alpha)} - (x + n + \delta) \tag{1}$$

where s is fixed saving rate, A and α are parameters of the Cobb-Douglas production function, x is the exogenous growth of technology, n is the population growth rate, and δ is the rate of depreciation.

(a) (10) Using a log-linear approximation of (1) in the neighborhood of the steady state we obtain

$$\gamma_{\hat{k}} = d\ln(\hat{k})/dt \cong -\beta[\ln(\hat{k}/\hat{k}^*)].$$

What is the expression for β?

(b) (10) What is σ convergence?

(c) (10) Discuss in words the relationship between β and σ convergence.

4. (50 points + 20 BONUS points) Consider the model with a variety of producer products we had in class with the production function for firm i given by

$$Y_i = AL_i^{1-\alpha} \sum_{j=1}^{N} X_{ij}^{\alpha}$$

where $0 < \alpha < 1$, Y_i is output, L_i is labor input, and X_{ij} is the employment of the jth type of specialized intermediate (nondurable) output.

(a) (10) Is a new type of product j a direct substitute or a direct complement with the types that already exists? [Hint: A product j substitutes for a product j' if it reduces the marginal product of $X_{j'}$. A product j complements a product j' if it raises the marginal product of $X_{j'}$.]

(b) (20) Derive the final good's sector demand for labor and intermediate goods as a function of the prices w and P_j.

(c) (20) We assume that the inventor of good j retains a perpetual monopoly right over the production and sale of the good, X_j, that uses his or her design. Suppose that, once invented, an intermediate good of type j costs one unit of final product, i.e. the marginal cost is equal to one. Specify the problem of the monopoly producer. What is the monopoly price P_j and the aggregate quantity produced of each good X_j?

(d) (*10*) Write the present value of the returns for a monopoly firm j from discovering the jth intermediate good $V(t)$. Suppose that the cost to create a new type of product is fixed at η units of final goods. Explain what would happen if $V(t) > \eta$ or if $V(t) < \eta$ and that $V(t) = \eta$ must hold. Using the last condition calculate the interest rate r.

(e) (*10*) Explain in words the nature of market failures (inefficiencies) that emerge from decentralized competition in the model. Describe in words at least one policy instrument which can eliminate the inefficiencies and explain why.