

## Final Examination

Read everything carefully before you start! The exam is worth of 180 points altogether.  
GOOD LUCK!

1. (35 points) Choose one of AK models we had in class, except the simplest one with the firm production function  $Y = AK$ . Write and describe the production function of a typical firm and explain what are the assumptions that eliminate the tendency for diminishing returns. Explain in words the nature of market failures (inefficiencies) that emerge from decentralized competition in the model. What sorts of policies could be enacted by a government in order to correct these market failures? Are there any scale effects in the model? Explain why or why not.
2. (25 points) Assume the Solow growth model with the rate of the exogenous growth of technological progress  $\dot{A}/A = g$  and the rate of population growth  $\dot{L}/L = n$  described by

$$\dot{K} = sK^\gamma (AL)^{1-\gamma} - \delta K$$

where  $0 < \gamma < 1$ ,  $s$  is fixed saving rate, and  $\delta > 0$  is the rate of depreciation.

- (a) (10) Derive the equation for the growth rate of capital per effective labor  $\gamma_{\hat{k}}$ , where  $\hat{k} = K/(AL)$ . Derive the steady-state condition for the level of capital per effective labor and the level of output per effective labor. What is the growth rate of output at steady state?
- (b) (10) Using a log-linear approximation of the growth equation for  $\hat{k}$  from (2a) in the neighborhood of the steady state we obtain

$$\gamma_{\hat{k}} = d \ln(\hat{k})/dt \cong -\beta[\ln(\hat{k}/\hat{k}^*)].$$

What is the expression for  $\beta$ ?

- (c) (5) What is  $\sigma$  convergence?

3. (120 points) Consider the model with a variety of producer products similar to which we had in class where

- the representative household solves the following problem

$$\max \int_0^{\infty} \ln c_t e^{-(\rho-n)t} dt$$

s.t.

$$c_t + \dot{v}_t = w_t + (r_t - n)v_t$$

with  $v$  denoting its asset-holding in the monopolistic firms producing intermediate goods; The initial conditions on the size of the household  $L_0 = 1$  and on the asset-holding  $v_0 > 0$  (notice that we assume here population growth  $n > 0$ );

- the production of the final good in a perfectly competitively firm  $i$  is given by

$$Y_i = L_i^\beta \sum_{j=1}^A X_{ij}^{1-\beta}$$

where  $0 < \beta < 1$ ,  $Y_i$  is output,  $L_i$  is labor input,  $X_{ij}$  is the employment of the  $j$ th type of specialized intermediate output, and  $A$  is the number of intermediate goods;

- the invention of a new type of intermediate nondurable good has a constant fixed cost of  $\phi > 0$  of final goods;
  - the monopolistic producer of an intermediate (nondurable) good  $j$ , who owns a perpetual patent on how to produce it, transforms one unit of the final good into one unit of the intermediate good  $j$ .
- (a) (10) Set up the Hamiltonian for the household problem, specify the control and the state variables, and derive the first-order conditions.
- (b) (10) Derive the final good's sector demand for labor and intermediate goods as a function of the prices  $w$  and  $P_j$ , where  $w$  is the price of labor and  $P_j$  is the price of intermediate good  $j$ .
- (c) (10) Specify the problem of the monopoly producer of intermediate good  $j$ . What is the monopoly price  $P_j$  and the aggregate quantity produced of each good  $X_j$ ?

- (d) (10) Calculate the profit of intermediate firm  $j$ . Write down the expression for the present discounted value,  $V_t$ , of the returns of monopoly firm  $j$  from producing the  $j$ th intermediate good. What is the relation between  $V_t$  and the cost of invention under the assumption of free-entry into the research sector?
- (e) (5) What is the equilibrium relation between the representative asset holding  $v_t$  and the value of the firm  $V_t$ ?
- (f) Consider first the case when the population growth  $n = 0$ .
- i. (10) Derive the expression for the interest rate  $r$  and prove that it is constant.
  - ii. (10) Derive and explain the Euler equation. Derive the formula for the growth rate of aggregate output.
  - iii. (10) Explain the nature of market failures that emerge from the decentralized model.
  - iv. (15) Write down the social planner problem for this economy under the condition that the social planner assumes that I-goods enters the production function for the final goods symmetrically. What are the state and control variables for this problem? Derive the first-order conditions. What is the social return on savings/investment?
  - v. (10) Propose and derive the government policy which will allow the market economy to reach the social optimum. Write down the appropriate government and household budget constraints for this case.
- (g) Now consider the case when the population growth  $n > 0$ .
- i. (10) Show that the interest rate  $r_t$  is not constant. Derive the expression for it.
  - ii. (10) What are the implications of the positive population growth for the behavior of the economy? (You don't have to make any mathematical derivations.)