CERGE-EI
Course: Macroeconomics III
Term: Summer 2007
Instructor: Michal Kejak
TA: Katarína Kotovová

Exercise Session 8
July 11, 2007

Problem 1. *Romer's model with modified production function in R&D.*[1]

Consider Romer (1990) version of the model with a variety od producer products, with the production function for firm i given by

$$Y_i = L_{Y,i}^{\beta} \sum_{j=1}^{A} X_{ij}^{1-\beta}$$

where $0 < \beta < 1$, Y_i is output, $L_{Y,i}$ is labor input, and X_{ij} is the employment of the jth type of specialized intermediate (nondurable) output.

1. Derive the final good's sector demand for labor and intermediate goods as a function of the prices w and P_j.

2. We assume that the inventor of good j retains a perpetual monopoly right over the production and sale of the good, X_j. As in class, one unit of final good combined with a design can be transformed into one unit of intermediate good. Solve the profit maximization problem for an intermediate goods firm that owns a patent and show that all intermediate goods sell for the same price. What is the instantaneous profit π earned by an I-firm as a function of labor used in the final-good sector?

3. Assume that the production function in R&D sector for discovering new ideas is more general than we had in class: the quantity of labor necessary to invent a new product is
$$\hat{\eta} = \frac{\eta}{L_R^{\lambda-1} A^{\epsilon}}$$
capturing the externality effects of the stock of ideas A, and the total labor searching for new ideas L_R. Assume that $\epsilon < 1$ and $0 < \lambda < 1$. Assume further that the population L grows at rate $n = \frac{\dot{L}}{L}$.

   (a) Explain the meaning of $\dot{A} = \frac{L_R}{\hat{\eta}}$ and interpret the meaning of $\hat{\eta}$. Using the above general production function specify the rate of technological change $\frac{\dot{A}}{A}$ as a function of the labor used in R&D sector and of the level of technology. Derive the expression for the rate of technological change along the balanced growth path (BGP). Is this model a true endogenous growth model? Why or why not? [Hint: Use the fact that along BGP $(\frac{\dot{L}_R}{L_R})^* = n$.]

[1] This problem is taken from Macro II final exam, 2006.

(b) Write down the formula for the present discounted value of a blueprint for an I-good $V(t)$. Show that it implies the arbitrage condition $r = \frac{\pi(t)}{V(t)} + \frac{\dot{V}(t)}{V(t)}$. Explain the meaning of the arbitrage condition. Prove that the profit grows at rate $\frac{\dot{\pi}(t)}{\pi(t)} = n$. Using this and the arbitrage condition along a BGP derive the expression for the present value $V^*(t)$ along the BGP.

(c) Explain the meaning of free entry in R&D sector. Write down the expression which captures free entry into R&D sector for this economy.

(d) Assume that the household divides his unit of time endowment between the working time into R&D, l_R, and the working time in the final goods sector, l_Y. So $l_R + l_Y = 1$ and the aggregate labor in the R&D and the final good sector are $L_R = l_R L$ and $L_Y = l_Y L$, respectively. Write down the household problem for this economy. What are the state and control variables for this problem?

(e) Explain in words the nature of all market failures (inefficiencies) that emerge from the decentralized model.

(f) Write down the social planner problem for this economy under the condition that social planner assumes that I-good enters the production for the final goods symmetrically. What are the state and control variables for this problem?