Macroeconomics III

CERGE

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Midterm Examination

Read everything carefully before you start! You are not required to solve BONUS questions! The exam is worth of 90 points alltogether without bonuses. GOOD LUCK!

1. (80 points) Consider the standard Ramsey model with an infinite horizon model with L representative households and the following characteristics: CARA (Constant Absolute Risk Aversion) utility function $u(c) = -\frac{1}{\alpha}e^{-\alpha c}$ with $\alpha > 0$, Cobb-Douglas production function with capital and labor, zero population growth and zero exogenous growth of technology. The government consumes an amount G_t that is fully funded by a tax on capital income. The tax rate, τ, is fixed; government consumption G_t is set to be equal to the aggregate tax revenues in period t ($G_t = \tau r_t K_t$).

   (a) (10) Show that the chosen utility function satisfies all the standard conditions on utility function. Derive an expression for the elasticity of the marginal utility with respect to consumption.

   (b) (15) Set up a representative household's optimization problem. What are the state and control variables?

   (c) (10) Derive the first order conditions.

   (d) (10) Derive and explain the Euler equation.

   (e) (10) Derive steady state capital per capita in terms of the parameters of the model. Explain the effect of the tax rate τ on the steady state capital per

capita. What is the effect of τ on the growth rate of consumption: at steady state and out of steady state?

(f) (15) Suppose that the economy is initially in a steady state with government policy $\tau > 0$ and suddenly there is an unexpected, permanent abolition of government spending so $\tau = 0$. Show the effects of such government policy on the household behavior in the phase plane (c, k) and also the time paths of consumption and capital. Analyze the case when the government policy had to be postponed but households are informed about it. Show the effects of such situation on the agents behavior.

(g) (10) Assume now that the government suddenly and permanently will not cancell the capital tax collection but rather it will start returning the tax revenue back to households in a lump-sum fashion. How will change the household budget constraint? Show the effects of such government policy on the household behavior in the phase plane (c, k) and also the time paths of consumption and capital.

2. (10) What is the No-Ponzi-Game condition? Explain.

1. *An "AK" Model with Physical and Human Capital.* Consider the following aggregate production function:

$$Y = K^{\alpha}(hL)^{1-\alpha},$$

where $h = H/L$ is human capital per person, Y is output, and K is physical capital. Accumulation of the capital inputs is given by

$$\begin{aligned} \dot{K} &= s_K Y - \delta K \\ \dot{H} &= s_H Y - \delta H \end{aligned}$$

where s_K and s_H are constant and exogenously given. The labor force, L, grows at the exogenous growth rate n.

(a) What is the per capita growth rate of the economy along the balanced growth path?

(b) Show that along the balanced growth path the production function can be written as $Y = AK$ where A is constant. What is the value of A?

(c) Discuss briefly the "story" one would tell to construct a decentralized, competitive equilibrium for this economy. Are there any externalities needed? Will the decentralized equilibrium be socially optimal if we endogenize the determination of s_K and s_H?

(d) Discuss the effect of imposing the inequality restrictions $I_K \geq 0$ and $I_H \geq 0$.