

CERGE-EI  
Course: Macroeconomics III  
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Exercise Session 4  
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**Problem 1.** *The OLG Model.* Consider a two-period overlapping generation (OLG) economy with young agents, who provide inelastically labor services in exchange for wages, consume and save, and with old agents, who consume all their wealth composed of saving and earned interest. The population grows at the rate  $n > 0$ . Let utility be logarithmic  $U_t = \ln c_{1t} + \beta \ln c_{2,t+1}$  with discount factor  $0 < \beta < 1$ , where  $c_{1t}$  is the consumption of young at time  $t$ , and  $c_{2,t+1}$  is the consumption of old at time  $t + 1$ . Let the production side be composed of many competitive firms endowed with technology  $\tilde{A}K_t^\alpha L_t^{1-\alpha}$  where  $K_t$  and  $L_t$  are capital and labor services inputs, respectively, and  $\tilde{A} > 0$  is constant, and  $0 < \alpha < 1$ . Assume that the capital stock does not depreciate.

1. Write down the optimization problem of an agent who lives for two periods and maximizes his lifetime utility.
2. How does the agent's intertemporal budget constraint look like?
3. What are the agent's optimal savings,  $s_t$ , and consumption when young,  $c_{1t}$  and when old,  $c_{2,t+1}$  as functions of factor prices?
4. Derive the equation for the accumulation of capital per worker in this economy and compute the factor prices in this economy.
5. Show the transition of the economy from the initial level of capital  $k_0 > k^*$  in the  $(k_t, k_{t+1})$ -plane where  $k^*$  is the steady state level of capital. What is the rate of growth of capital per worker  $k$  on the balanced growth path?

**Problem 2.** *The OLG Model with Government.* Let introduce the government into the economy from Problem 1 above. The role of the government is to tax labor income and capital income by the same tax rate  $0 < \tau < 1$  and to tax consumption by the tax rate  $0 < \tau_c < 1$ . All the government tax receipts in given period are equal to the unproductive government spending at that period.

1. Write down the optimization problem of an agent who lives for two periods and maximizes his lifetime utility.
2. How does the agent's intertemporal budget constraint look like now?
3. Write down the government budget constraint. Is the budget balanced at each period?
4. What are the agent's optimal savings,  $s_t$ , and consumptions when young,  $c_{1t}$  and when old,  $c_{2,t+1}$  as functions of factor prices? Explain in words the effect of different taxes on the agent's decision.

5. Derive the equation for the accumulation of capital per worker in this economy.
6. Assume now that the economy is initially at the steady state when the government unexpectedly and permanently decreases the income tax rate to the level  $\tau'$ , i.e.  $0 < \tau' < \tau$ . Analyze the adjustment of the economy to this government policy change in  $(k_t, k_{t+1})$ -plane. Discuss the effect of the tax rate change on the steady state levels of consumption and savings. What is the rate of growth of capital per worker  $k$  on the new balanced growth path?
7. How will adjustment of the economy change when the economy is currently at time  $t$ , at steady state and the agents receive the information that the income tax rate will not be decreased immediately but rather in the next period at  $t + 1$ ? Discuss also the effect of the new information on the behavior of current variables.
8. Let us assume now that the shift parameter  $\tilde{A}$  in the production function captures the externality effect of the capital, i.e.  $\tilde{A} = Ak^\beta$  and  $\alpha + \beta = 1$ .
  - Compute the factor prices. Analyze the behavior of the economy in  $(k_t, k_{t+1})$ -plane. What is the rate of growth of capital per worker  $k$  on the balanced growth path?
  - What happens in this economy when the government unexpectedly and permanently decreases the income tax rate to the level  $\tau'$ , i.e.  $0 < \tau' < \tau$ . What is the rate of growth of capital per worker  $k$  on the new balanced growth path?