

CERGE-EI
Course: Macroeconomics III
Term: Summer 2007
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Exercise Session 1
May 16, 2007

Problem 1. Short review of Hamiltonians.

Problem 2. Show that the the following first order differential equation

$$\frac{d}{dt} \ln \hat{y} = -\beta(\ln \hat{y} - \ln \hat{y}^*)$$

has solution in a form

$$\ln \hat{y}(s) = (1 - e^{-\beta s}) \ln \hat{y}^* + e^{-\beta s} \ln \hat{y}(0).$$

Problem 3¹. *Solow model with natural resources.* At least since Malthus, some have argued that the fact that some factors of production are available in finite supply (notably land and natural resources) must eventually bring growth to a halt. This prediction is known as the Malthusian trap due to pessimistic beliefs Malthus had.

Let the production function be $Y = K^\alpha (AL)^\beta T^{1-\alpha-\beta}$, where T is the amount of land. Assume $\alpha > 0$, $\beta > 0$, and $\alpha + \beta < 1$. The factors of production evolve according to $\dot{K} = sY - \delta K$, $\dot{A} = gA$, $\dot{L} = nL$, and $\dot{T} = 0$.

1. Does this economy have a unique and stable balanced growth path? That is, does this economy converge to a situation in which each of Y , K , L , A , T are growing at a constant rates? If so, what are the those growth rates?
2. In light of your answer, does the fact that the stock of the land is constant imply that the permanent growth is not possible?
3. Under what circumstances will Y/L grow over time, as opposed to shrinking steadily? Give your answer in terms of α , β , n and g .
4. Was Malthus right? Discuss the results of the model above and their resemblance with Malthus predictions. How can economy escape from the Malthusian trap?

¹This problem is taken from MIT OpenCourseWare site.